Homework Assignment II

Reading Assignment: Lecture Notes; Kerns-Irwin Chapter 3.

1. Consider the resistive circuit in a diamond topology on the left of the figure below. This is commonly known as the *Wheatstone Bridge*, often used in electrical measurement applications. Note that this circuit can be redrawn as shown on the right side.



Figure 1: The Wheatstone Bridge.

- (a) Find $v_{ac}(t)$ the voltage across the resistor R_2 and $v_{bc}(t)$ the voltage across the resistor R_4 as a function of the source voltage $v_S(t)$.
- (b) Find $v_O(t)$ the voltage across nodes a and b as a function of the source voltage $v_S(t)$ given that $R_1 = R_4$ and $R_2 = R_3 = 2R_1$. *Hint:* $v_{ab}(t) = v_{ac}(t) v_{bc}(t)$.
- (c) If the source voltage is $v_S(t) = 9\cos(4\pi t)$, find the period, frequency, average voltage V_{AVE} , and root-mean-squared voltage V_{RMS} for $v_O(t)$.
- (d) Sketch the instantaneous power $p_S(t)$ generated by the voltage source given the same set-up as in Part (b) and (c).
- (e) If the stock room only carries 1Ω resistors, design and sketch your resulting Wheatstone Bridge where $v_O(t)$ is always precisely $\frac{1}{6}v_S(t)$.
- (f) Suppose that the source voltage is $v_S(t) = 9\cos(4\pi t)$ and $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$. Find the Thevenin equivalent circuit looking into the 2 terminals *a* and *b*.
- (g) With the same set-up as in Part (f), find the Norton equivalent circuit looking into the 2 terminals *a* and *b*.

2. The circuits below models the voltage produced by the generator of a power plant and R_S models the resistive losses in the generator, wire, and transformers. Resistances R_1 , R_2 , and R_3 represent the various loads connected by a customer. Assume that $R_1 = R_2 = R_3 = R_S = R$.



Figure 2: An electrical network model with power plant and consumers.

- (a) Find the load voltage $v_L(t)$ as a function of the source voltage $v_S(t)$ for t < 0 (before the customer closes the switch and connects the third load).
- (b) Find the load voltage $v_L(t)$ as a function of the source voltage $v_S(t)$ for t > 0 (after the customer closes the switch and connects the third load).
- (c) Sketch the load voltage $v_L(t)$ and the source current $i_S(t)$ as functions of time assuming that $v_S(t)$ is a DC source with peak value of 150V.
- (d) Sketch the source current $i_S(t)$ as a function of time assuming that $v_S(t)$ is a 60Hz AC source with RMS value of 170V.
- (e) For the AC source above, how much energy does the voltage source at the power plant produce over one source period (cycle) before and after the switch is closed?
- (f) **Bonus:** What is the Thevenin and Norton equivalence (from the 2 right-most terminals) of this circuit if there are N connected customers in parallel?

Due date: September 23 in class