1. In many applications, it is desirable if we can represent a quantized value \( y \) with the property that the least significant bits of the input \( x \) in sign-magnitude representation are simply discarded should we need to reduce the data rate or storage requirements. Consider such a quantizer called a bit-plane or an embedded quantizer as shown in the figure below. Assume that the range of \( x \) is \((-16, 16)\).

(a) Find the decision boundaries, quantization stepsizes, and the quantized levels for the bit-plane quantizer when only the first 3 bits (the sign bit and two most significant bits) are kept. Sketch the quantizer’s input-output relationship.

(b) Find the set of reconstructed values \( \{\hat{x}_i\} \) using the centroid rule and sketch the quantization error signal \( q(x) \) for the case above.

(c) Describe the nature of this type of quantizer. Is it uniform or non-uniform? Midrise or midtread?

(d) If the quantizer keeps one more additional bit: one sign plus three most significant bits, what happens to the quantization stepsize?
2. Consider the 3-level mid-thread quantizer depicted in the figure below.

(a) Find the set of reconstructed values \( \{ \hat{x}_i \} \) using the centroid rule and sketch the quantization error signal \( q(x) \).

(b) What is the quantization noise power \( \sigma_q^2 \) for the 3-level quantizer in Part (a)?

(c) For the 2-level (1-bit) mid-rise quantizer below, find the set of reconstructed values \( \{ \hat{x}_i \} \) using the centroid rule and sketch the quantization error signal \( q(x) \).

(d) What is the quantization noise power \( \sigma_q^2 \) for the 2-level quantizer in Part (c)? Which quantizer is superior?

(e) For the 2-level (1-bit) mid-rise quantizer, suppose that we do not follow the centroid reconstruction rule and choose the following 2 reconstruction levels \( \{-A, A\} \) instead. Sketch the quantization noise \( q(x) \) for this case and recompute \( \sigma_q^2 \). How does the noise power compare to your answer in Part (d)?

Due date: November 16 in class