Department of Electrical and Computer Engineering
The Johns Hopkins University
520.137 Introduction to Electrical and Computer Engineering – Fall 2012

Homework Assignment VII

Reading Assignment: Kuc Chapter 4

1. In many applications, it is desirable if we can represent a quantized value y with the property that the least significant bits of the input x in sign-magnitude representation are simply discarded should we need to reduce the data rate or storage requirements. Consider such a quantizer called a bit-plane or an embedded quantizer as shown in the figure above. Assume that the range of x is $(-16, 16)$.

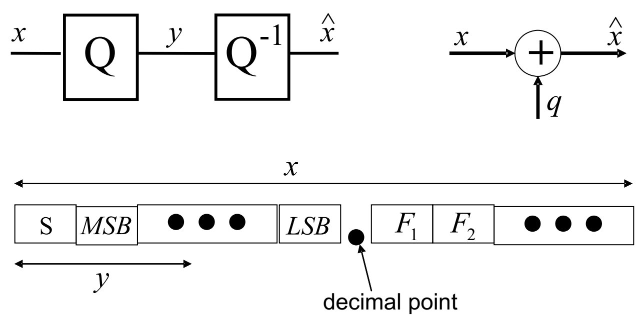

Figure 1: The bit-plane or embedded quantizer.

   (a) Find the decision boundaries, quantization stepsizes, and the quantized levels for the bit-plane quantizer when only the first 3 bits (the sign bit and two most significant bits) are kept. Sketch the quantizer's input-output relationship.

   (b) Find the set of reconstructed values $\{\hat{x}_i\}$ using the centroid rule and sketch the quantization error signal $q(x)$ for the case above.

   (c) Describe the nature of this type of quantizer. Is it uniform or non-uniform? Midrise or midtread?

   (d) If the quantizer keeps one more additional bit: one sign plus three most significant bits, what happens to the quantization stepsize?

2. Consider a uniform quantizer as follows where

$$y = \lceil x/2 \rceil.$$

   Reminder: $\lceil x \rceil$ = the smallest integer greater than or equal to x.

(a) Find the decision boundaries, quantization stepsize, and the quantized levels for the quantizer based on the *ceiling* operator. Sketch the quantizer's input-output relationship.

(b) Find the set of reconstructed values $\{\hat{x}_i\}$ using the centroid rule. Find the mathematical expression describing the reconstruction $\hat{x}$ as a function of the quantized signal y.

(c) Sketch the quantization error signal $q(x)$ from Part (a) and (b).

(d) Find the power of the quantization noise σ_q^2 for this quantizer.

(e) Suppose that the reconstruction values are now moved *toward zero* by 1 (moving left when $\hat{x} > 0$ and moving right when $\hat{x} < 0$). Re-sketch the quantization error signal $q(x)$ and compute the power of the quantization noise in this case.

3. Consider the 2-level 1-bit mid-rise quantizer depicted in the figure below.

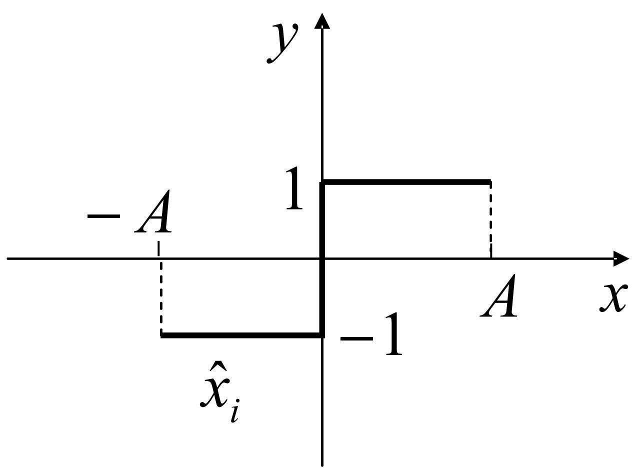

Figure 2: A 2-level 1-bit mid-rise quantizer.

(a) Find the set of reconstructed values $\{\hat{x}_i\}$ using the centroid rule and sketch the quantization error signal $q(x)$.

(b) What is the quantization noise power σ_q^2 for the 2-level quantizer in Part (a)?

(c) Suppose that we do not follow the centroid reconstruction rule and choose the following 2 reconstruction levels $\{-\frac{A}{4}, \frac{A}{4}\}$ instead. Sketch the quantization noise $q(x)$ for this case and recompute σ_q^2. How does the noise power compare to your answer in Part (b)?

Due date: **Nov. 14** in class