Department of Electrical and Computer Engineering
The Johns Hopkins University
520.137 Introduction to Electrical and Computer Engineering – Fall 2016

Homework Assignment VII

Reading Assignment: Kuc Chapter 4

1. Suppose that we would like to sample the following function

$$x(t) = 520\cos[5200\pi(t+1)] + 137\cos(1370\pi t + \frac{\pi}{8}) + \sin[2013\pi(t-1)].$$

   (a) Find the sampling rates such that aliasing is guaranteed to be avoided. What is the corresponding sampling period?

   (b) If the highest frequency component is removed from $x(t)$, what is now the Nyquist sampling rate and sampling period?

2. The audio signal in a telephone line has been band-limited to a maximum frequency of 3 kHz.

   (a) Determine the sampling rate at which the A/D converter must sample the signal in order to capture an accurate record of it.

   (b) Determine the time-interval spacing between digital samples if
      - we use Nyquist sampling rate as in Part (a)
      - we use 50% oversampling, that is the sampling rate is 50% higher than Nyquist rate
      - we use 50% undersampling, that is the sampling rate is 50% lower than Nyquist rate.

   (c) Suppose that the A/D converter uses 16 bits (2 bytes) to store each output digital sample. Determine the amount of memory required to store 10 seconds of speech for each of the following 3 cases:
      - we use Nyquist sampling rate as in Part (a)
      - we use 50% oversampling, that is the sampling rate is 50% higher than Nyquist rate
      - we use 50% undersampling, that is the sampling rate is 50% lower than Nyquist rate.

   (d) What is the corresponding communication bit-rate in kbps (kilo-bits per second) in each of the three cases above?

3. Consider a uniform quantizer as follows where

$$y = \lceil x/2 \rceil.$$

   Reminder: $\lceil x \rceil$ = the smallest integer greater than or equal to x.

   (a) Find the decision boundaries, quantization stepsize, and the quantized levels for the quantizer based on the *ceiling* operator. Sketch the quantizer's input-output relationship.

   (b) Find the set of reconstructed values $\{\hat{x}_i\}$ using the centroid rule. Find the mathematical expression describing the reconstruction $\hat{x}$ as a function of the quantized signal y.

(c) Sketch the quantization error signal $q(x)$ from Part (a) and (b).

(d) Find the power of the quantization noise σ_q^2 for this quantizer.

(e) Suppose that the reconstruction values are now moved *toward zero* by 1 (moving left when $\hat{x} > 0$ and moving right when $\hat{x} < 0$). Re-sketch the quantization error signal $q(x)$ and compute the power of the quantization noise in this case.

4. Consider the 2-level 1-bit mid-rise quantizer depicted in the figure below.

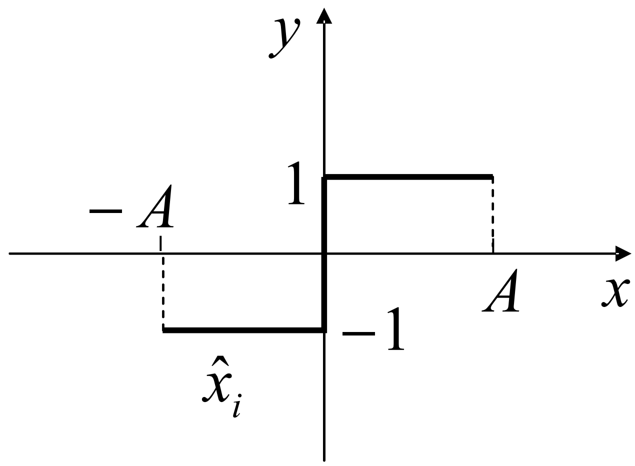

Figure 1: A 2-level 1-bit mid-rise quantizer.

(a) Find the set of reconstructed values $\{\hat{x}_i\}$ using the centroid rule and sketch the quantization error signal $q(x)$.

(b) What is the quantization noise power σ_q^2 for the 2-level quantizer in Part (a)?

(c) Suppose that we do not follow the centroid reconstruction rule and choose the following 2 reconstruction levels $\{-\frac{A}{4}, \frac{A}{4}\}$ instead. Sketch the quantization noise $q(x)$ for this case and recompute σ_q^2. How does the noise power compare to your answer in Part (b)?

Due date: **Nov. 11** in class