

Homework Assignment IX

Reading Assignment: Kuc Chapter 7

1. Consider the following (5, 2) ECC code which contains a mixture of parity and repetitive bits:

$$M_1M_2P_1P_2P_3$$

where the two repetitive bits are $P_1 = M_1$ and $P_2 = M_2$ whereas the parity bit is $P_3 = M_1 \oplus M_2$.

- (a) Construct a table of all valid code-words.
- (b) What is the minimum Hamming distance between the valid code-words? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?
- (c) Another (5, 2) ECC code is constructed as follows:

$$M_1M_2P_1P_2P_3$$

where $P_1 = M_1 \oplus M_2$, $P_2 = M_1M_2$, and $P_3 = M_1 + M_2$.

Again, construct a table of all valid code-words.

- (d) What is the minimum Hamming distance for this ECC code? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?
- (e) A purely repetitive (6, 2) ECC code is constructed by simply repeating each message bit twice.
Construct a table of all valid code-words for this (6, 2) repetitive code.
- (f) What is the minimum Hamming distance for this ECC code? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?
- (g) Among the 3 ECC codes explored in Part (a), (c) and (e), which one is the best? Justify briefly.

2. Consider the design of a Hamming-style (5, 3) parity-checking code. Each valid code-word has the following form:

$$X_3 X_2 X_1 P_2 P_1$$

where the 2 parity bits are $P_2 = X_2 \oplus X_3$ and $P_1 = X_1 \oplus X_2$.

- Construct a table of all valid code-words.
 - What is the minimum Hamming distance between the valid code-words? What is the maximum Hamming distance for this code?
 - How many bit error(s) can we detect in this case? How many bit error(s) can we correct?
 - Can we improve the detection/correction capability by adding one additional parity bit as follows: $P_3 = X_3 \oplus X_1$? Why or why not?
3. Consider the following (7, 4) error-resilient code below:

$$M_1 M_2 M_3 M_4 P_1 P_2 P_3$$

where

$P_1 = M_1 \oplus M_3$ is the parity-check bit for the odd-indexed message-bit,

$P_2 = M_2 \oplus M_4$ is the parity-check bit for the even-indexed message-bit,

$P_3 = P_1 \oplus P_2$ offers a second layer of protection on the two parity bits.

- Construct a table of all valid code-words. How many valid code-words are there? How many possible invalid code-words are there?
- What is the maximum Hamming distance between the valid code-words? What is the minimum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?
- Design the error-detection circuit for this redundant code using basic gates.
- If we replace the third parity-bit as follows

$$P_3 = M_1 \oplus M_2,$$

and we add a fourth parity bit

$$P_4 = M_3 \oplus M_4,$$

can we improve the error detection capability? Can we improve the error correction capability improved? Explain.

- Implement the modified encoder in Part (d) with basic gates.