Department of Electrical and Computer Engineering
The Johns Hopkins University
520.443 Digital Multimedia Coding and Processing – Spring 2007

Homework Assignment I

1. Let X denote a random variable distributed on the set $\{x_1, x_2, \ldots, x_N\}$ with associated probabilities $\{P_1, P_2, \ldots, P_N\}$. Let Y be another random variable defined on the same set but distributed uniformly.

   (a) Show that
   $$H(X) \;\leq\; H(Y).$$
   Hint : First prove the inequality $\ln w \leq w - 1$ with equality for $w = 1$, then apply this inequality to $\sum_{n=1}^{N} P_n \ln \frac{1/N}{P_n}$.

   (b) Show that the entropy H of $\mathcal{X}$ satisfies
   $$0 \;\leq\; H \;\leq\; \log N.$$

   (c) Find the necessary and sufficient conditions under which equality holds.

2. Design a Huffman code for a source with n output symbols and corresponding probabilities $\{\frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{2^{n-1}}, \frac{1}{2^{n-1}}\}$. Show that the average codeword length for such a source is equal to the source entropy.

3. Consider a source with 3 symbols and corresponding probabilities 0.95, 0.04, and 0.01.

   a. Find the entropy of this source.

   b. Design the Huffman code for this source. Compare your average codeword length with the entropy.

   c. Combine two symbols and re-design your Huffman code. Again, compare your average codeword length to the entropy. Is there any improvement?

4. **Computer Assignment**. Go to the course webpage

http://thanglong.ece.jhu.edu/Course/443/

Click on Lecture Notes and download the QCIF sequence named `glasgow100.qcif`. Please also read the `format.txt` file to understand how a typical raw YCrCb video sequence is stored. Each pixel is represented by 1 byte (`unsigned char` in C) and takes on values $0 - 255$. You can play these sequences using `splay.c`, `YUVplayer.exe`, or `CIFplayer.exe` – all available on the course webpage. A nice and simple image viewer `IrfanView` is also available. The file `utility.c` contains some simple routines that you may find useful.

a. Write a computer program to extract any luminance frame and output it as a `pgm` image given the video data and the frame index as inputs. The `pgm` format is a raw gray-scale image file with the following header:
P5
width height
255
where width and height are 176 and 144, respectively, in QCIF format. `IrfanView` can open `pgm` files.

b. Write a program to compute the entropy, the mean-square error, the peak signal-to-noise ratio, the mean absolute difference, and the maximum pixel error between any two given images or video frames.

c. Use the program above to compute the mean-square error between frame 0 and each of frames 1 to 30 of the *Glasgow* sequence. Plot the MSE's as a function of time (index) difference. What do you observe from your plot?

Due date: **Feb. 14** in class