Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2018

Homework Assignment I

Reading Assignment: lecture notes, Strang-Nguyen Sections 2.1 – 2.3, 3.1 – 3.3

1. Let $G(z) = \sum_{n=0}^{N} g[n]z^{-n}$ be a real-coefficient order-N FIR low-pass filter whose impulse response is $g[n]$ and $g[0], g[N] \neq 0$. Define the following three filters:

$$H_1(z) \;=\; z^{-N}G(z^{-1}); \qquad H_2(z) \;=\; G(-z); \qquad H_3(z) \;=\; z^{-N}G(-z^{-1}).$$

   (a) Find the impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ in term of $g[n]$.

   (b) If $g[n]$ is even-length and symmetric, which type of filter is $h_1[n]$, $h_2[n]$, and $h_3[n]$?

   (c) Find the magnitude responses of $h_1[n]$, $h_2[n]$, and $h_3[n]$ in term of $|G(e^{j\omega}|$. If $G(z)$ is the perfect low-pass filter with cut-off frequency ω_C, sketch the frequency responses of $H_1(z)$, $H_2(z)$, and $H_3(z)$.

   (d) If z_0 is a zero of $G(z)$, find the corresponding zeros of $H_1(z)$, $H_2(z)$, and $H_3(z)$.

2. Prove that an FIR filter $H(z)$ of order N has real coefficients $h[n]$ if and only if its roots are either real or appear in conjugate pairs. More precisely, the right-hand side means that: if z_n is a root of $H(z)$, then either z_n is real or z_n^* is also a root of $H(z)$.

3. Prove that the amplitude response of an FIR filter $H(z)$ of order N with real coefficients $h[n]$ has symmetry.

Due date: **Friday September 14** in class