Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2012

Homework Assignment I

Reading Assignment: lecture notes, Strang-Nguyen Sections 2.1 – 2.3, 3.1 – 3.3

1. Let $G(z) = \sum_{n=0}^{N} g[n]z^{-n}$ be a real-coefficient order-N FIR low-pass filter whose impulse response is $g[n]$ and $g[0], g[N] \neq 0$. Define the following three filters:

$$H_1(z) = z^{-N}G(z^{-1}); \quad H_2(z) = G(-z); \quad H_3(z) = z^{-N}G(-z^{-1}).$$

   (a) Find the impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ in term of $g[n]$.

   (b) If $g[n]$ is even-length and symmetric (Type-II real linear phase), which type of filter is $h_1[n]$, $h_2[n]$, and $h_3[n]$?

   (c) Find the magnitude responses of $h_1[n]$, $h_2[n]$, and $h_3[n]$ in term of $|G(e^{j\omega}|$. If $G(z)$ is the perfect low-pass filter with cut-off frequency ω_C, sketch the frequency responses of $H_1(z)$, $H_2(z)$, and $H_3(z)$.

   (d) If z_0 is a zero of $G(z)$, find the corresponding zeros of $H_1(z)$, $H_2(z)$, and $H_3(z)$.

2. Prove that the downsampler and the upsampler are linear time-varying systems.

3. An input signal $x[n]$ with triangular frequency spectrum is passed through a 2-channel maximally-decimated filter bank with ideal filters as depicted in Figure 1.

   Sketch the signal spectrum at every node. Show graphically that perfect reconstruction can be achieved.

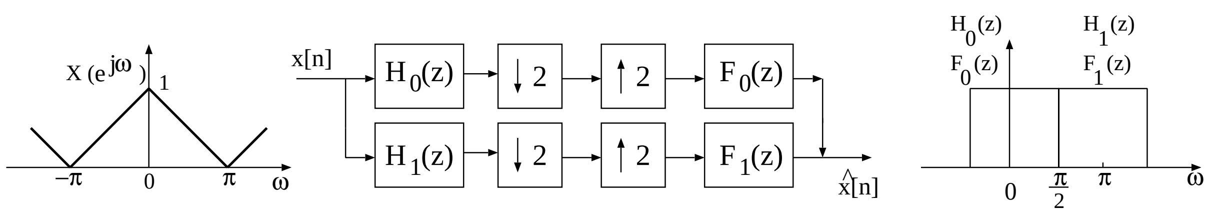

Figure 1: Two-channel filter bank with ideal filters.

Due date: **Wed. September 19** in class