Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2012

Problem Set III

Go to the course website and download the wavelet software package. You should find in the package Matlab codes for the 1D as well as the 2D wavelet transform.

1. Write a Matlab function to test for perfect reconstruction of any four given filters $H_0(z)$, $H_1(z)$, $F_0(z)$, $F_1(z)$. You should test the aliasing cancellation and the distortion elimination condition.

2. Let $p_0[n] = \frac{1}{2048}[\,-5\;\;0\;\;49\;\;0\;\;-245\;\;0\;\;1225\;\;2048\;\;1225\;\;0\;\;-245\;\;0\;\;49\;\;0\;\;-5\,]$.

   (a) Is $P_0(z)$ a maxflat halfband filter? Find and plot its zeros.

   (b) Distribute the zeros of $P_0(z)$ such that $H_0(z)$ is real, linear-phase, and of length 6, whereas $F_0(z)$ is real, linear-phase, and of length 10. Find all possible solutions.

   (c) For each solution above, plot the time and frequency responses of all 4 filters. Verify perfect reconstruction using your Matlab function in Problem 1.

   (d) For each of your solution, compute the 4-level discrete wavelet transform of the *Boat* image. Reconstruct the image from only 10 % largest coefficients (the rest is set to zero). Compare the reconstructed image quality and the peak signal-to-noise ratio (PSNR) with respect to the original image.

   (e) Now factor the same filter $P_0(z)$ into odd-length real linear phase filters $H_0(z)$ and $F_0(z)$. Find all possible solutions that yield 9/7 taps.

   Repeat Part (c) and (d) in Problem 3 with your new odd/odd systems.

   (f) Find all possible orthogonal solutions. Verify orthogonality and shift-orthogonality. Again, repeat Part (c) and (d) above.

Due date: **Friday Oct. 5** in class