Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2017

Homework Assignment III

Reading Assignment: Lecture Notes; Strang-Nguyen Chapter 3, Section 4.1-4.2;

1. Let $P_0(z) = (1 + z^{-1})^6\ Q(z)$, find the 4-th degree symmetric polynomial $Q(z)$ that makes $P_0(z)$ halfband. Find and plot all the roots of the resulting halfband filter.

2. Consider the 2-channel filter bank depicted in Figure 1 with

$$\begin{aligned} H_0(z) &= a + bz^{-1} + bz^{-2} + az^{-3} \\ H_1(z) &= c + dz^{-1} - dz^{-2} - cz^{-3}. \end{aligned}$$

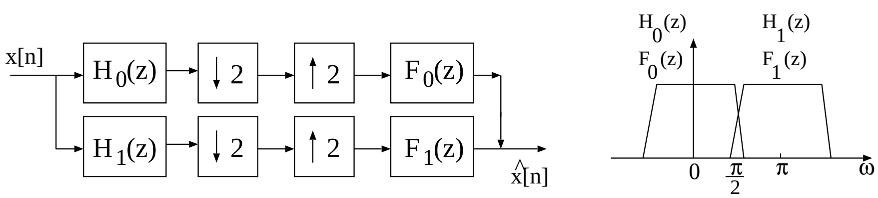

Figure 1: Two-channel maximally decimated filter bank.

   (a) Find $F_0(z)$ and $F_1(z)$ such that aliasing can be canceled.
   (b) Find the analysis polyphase matrix $\mathbf{H}_p(z)$. Prove that $\mathbf{H}_p(z)$ has symmetry:

$$\mathbf{H}_p(z) \ = \ z^{-1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{H}_p(z^{-1}) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

   (c) Find the determinant of $\mathbf{H}_p(z)$.
   (d) Based on the determinant of $\mathbf{H}_p(z)$, find the constraints on $\{a, b, c, d\}$ that yield perfect reconstruction with FIR synthesis filters.
   (e) Find the causal FIR synthesis polyphase matrix $\mathbf{F}_p(z)$? What are the corresponding synthesis filters? Are your answers consistent with those in Part a?
   (f) Imposing one more zero at $\omega = \pi$ on $H_0(z)$ by setting its first derivative to zero at $z = -1$. What are the four resulting filters now?

Due date: **Friday 09/29/2017** in class