Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2012

Problem Set IV

1. Write Matlab functions to compute and plot the scaling function $\phi(t)$ and the wavelet function $\psi(t)$ given two FIR filters $h_0[n]$ and $h_1[n]$.

2. Let $p_0[n] = \frac{1}{2048}[\,-5 \;\; 0 \;\; 49 \;\; 0 \;\; -245 \;\; 0 \;\; 1225 \;\; 2048 \;\; 1225 \;\; 0 \;\; -245 \;\; 0 \;\; 49 \;\; 0 \;\; -5\,]$ as in Problem Set IV.

   (a) For every real orthogonal solution, plot the associated scaling function $\phi(t)$ and the wavelet function $\psi(t)$. Any observation on how the zeros distribution affect the phase of the scaling and wavelet functions?

   (b) For every real 6/10-tap biorthogonal solution, plot the analysis as well as the synthesis $\phi(t)$ and $\psi(t)$. Repeat the exercise for every real 9/7-tap biorthogonal solution. Report your observations on any correlation between the smoothness of $\{\ \phi(t), \psi(t)\ \}$, the number of zeros at π of $\{\ h_0[n], f_0[n]\ \}$, and the quality of the reconstructed signals after quantization.

3. Consider a multiresolution analysis and the two-scale equation for $\phi(t)$ given in $\phi(t) = \sqrt{2}\sum_n h_0[n]\phi(2t-n)$. Assume that $\{\phi(t-n)\}$ is an orthonormal basis for $\mathcal{V}_0$ and the associated orthogonal filter bank with $h_0[n]$ and $h_1[n]$. Assume further that $0 < |\Phi(0)| < \infty$ and that $\Phi(\omega)$ is continuous at $\omega = 0$. Prove the following statements.

   (a) $\Phi(0) = \int \phi(t)dt = 1$.

   (b) $h_1[n] = \sqrt{2} < \phi(2t-n), \psi(t) >$.

   (c) $h_0[n] = \sqrt{2} < \phi(2t-n), \phi(t) >$.

   (d) $|\Phi(\omega)|^2 + |\Psi(\omega)|^2 \;=\; |\Phi(\frac{\omega}{2})|^2$.

   (e) $||h_0[n]|| = 1$. *Hint: Start with* $||\phi(t)||^2$.

Due date: **October 19** in class