Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2012

# Problem Set V

1. Demonstrate in Matlab the polynomial capturing ability of the discrete wavelet transform with certain number of vanishing moments. Experiment with your best 9/7 biorthogonal, 6/10 biorthogonal, and 8/8 orthogonal solution obtained from Problem Set IV. What are your observations in the biorthogonal cases?

   Now suppose that your polynomial signal is corrupted by white noise. Design a simple algorithm to recover the polynomials.

2. Suppose that $H_0(e^{j\omega})$ is an ideal low-pass filter and $H_1(e^{j\omega})$ is an ideal high-pass filter. Both filters have cut-off frequency at $\omega = \frac{\pi}{2}$ and are periodic with period $2\pi$.

   Find the frequency and time responses of the resulting scaling and the wavelet function, i.e., $\Phi(\omega)$, $\Psi(\omega)$, $\phi(t)$, $\psi(t)$.

3. Suppose that we have $K$ vanishing moments in an orthonormal wavelet system, prove that

   (a) $\{\frac{d^k}{dz^k}H_1(z)\}|_{z=1} = 0, \quad$ for $k = 0, 1, \ldots, K-1$.

   (b) $\sum_n n^k h_1[n] = 0, \quad$ for $k = 0, 1, \ldots, K-1$.

   (c) $\int t^k \psi(t)dt = 0, \quad$ for $k = 0, 1, \ldots, K-1$. *Hint:* The Fourier tranform of $t^k\psi(t)$ is $(-j)^k\Psi^{(k)}(\omega)$.

Due date: **Friday Oct. 26** in class