

## Problem Set VI

1. Problem 4.5-7
2. Problem 4.5-14
3. Problem 9.2-11
4. You are given a 2-channel perfect reconstruction filter bank ( $\hat{x}[n] = x[n-\ell]$ ) with filters  $H_0(z), H_1(z), F_0(z), F_1(z)$  and associated polyphase matrices  $\mathbf{H}_p(z)$  and  $\mathbf{F}_p(z)$ . Now, consider the addition of a lifting step

$$\begin{bmatrix} 1 & 0 \\ -P(z) & 1 \end{bmatrix}$$

as shown in Figure 1 below.

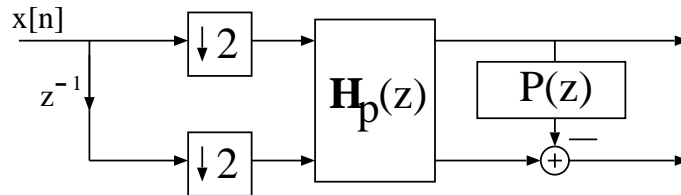


Figure 1: A polyphase construction with lifting step.

- (a) Draw the corresponding synthesis bank and find the new polyphase matrices  $\mathbf{H}'_p(z)$  and  $\mathbf{F}'_p(z)$ .
- (b) Show that the new analysis filters are

$$H'_0(z) = H_0(z) \quad \text{and} \quad H'_1(z) = H_1(z) - P(z^2)H_0(z).$$

- (c) Find the new synthesis filters and the time-domain relationships between new and the old filters.
- (d) Show that the addition of the lifting step leaves the analysis scaling function  $\phi(t)$  intact but yields a new analysis wavelet function

$$\psi'(t) = \psi(t) - \sqrt{2} \sum_k p[k] \phi(t - k).$$

- (e) How does the addition of the lifting step alter the degree of regularity in the analysis and the synthesis bank?

Due date: **Nov. 2** in class