

Problem Set VI

1. Consider a multiresolution analysis and the two-scale equation for $\phi(t)$ given in $\phi(t) = \sqrt{2} \sum_n h_0[n] \phi(2t - n)$. Assume that $\{\phi(t - n)\}$ is an orthonormal basis for \mathcal{V}_0 and the associated orthogonal filter bank with $h_0[n]$ and $h_1[n]$. Assume further that $0 < |\Phi(0)| < \infty$ and that $\Phi(\omega)$ is continuous at $\omega = 0$. Prove the following statements.
 - (a) $\Phi(0) = \int \phi(t) dt = 1$.
 - (b) $h_1[n] = \sqrt{2} \langle \phi(2t - n), \psi(t) \rangle$.
 - (c) $h_0[n] = \sqrt{2} \langle \phi(2t - n), \phi(t) \rangle$.
 - (d) $|\Phi(\omega)|^2 + |\Psi(\omega)|^2 = |\Phi(\frac{\omega}{2})|^2$.
 - (e) $\|h_0[n]\| = 1$. *Hint: Start with $\|\phi(t)\|^2$.*
2. Demonstrate in Matlab the polynomial capturing ability of the discrete wavelet transform with certain number of vanishing moments. Experiment with your best 9/7 biorthogonal, 6/10 biorthogonal, and 8/8 orthogonal solution obtained from Problem Set IV. What are your observations in the biorthogonal cases?

Now suppose that your polynomial signal is corrupted by white noise. Design a simple algorithm to recover the polynomials.

Due date: **Friday Nov. 10** in class