

Homework Assignment I

Reading Assignment: Wired Magazine article by Jordan Ellenberg at the following URL:
http://www.wired.com/magazine/2010/02/ff_algorithm/all/1

1. Recall from the class that for a matrix $A = (a_{jk}) \in C^{m \times N}$, the operator norm of A from l_p into l_p is defined as:

$$\|A\|_{p \rightarrow p} := \max_{\|x\|_p=1} \|Ax\|_p$$

Show that:

- (a) $\|A\|_{1 \rightarrow 1} = \max_{k \in [N]} \sum_{j=1}^m |a_{jk}|$
 - (b) $\|A\|_{\infty \rightarrow \infty} = \max_{j \in [m]} \sum_{k=1}^N |a_{jk}|$
 - (c) $\|A\|_{2 \rightarrow 2} = \sigma_{\max}(A) = \sqrt{\lambda_{\max}(A^*A)}$ where $\sigma_{\max}(A)$ denotes the largest singular value of A and $\lambda_{\max}(A^*A)$ is the largest eigenvalue of A^*A .
 - (d) If A is Hermitian (i.e. $A^* = A$) then, show that
 - i. $\|A\|_{2 \rightarrow 2} \leq \|A\|_{1 \rightarrow 1}$
 - ii. $\|A\|_{2 \rightarrow 2} = \sup_{\|x\|_2=1} |\langle Ax, x \rangle|$
2. Let V be a normed vector space with an inner product $\langle \cdot, \cdot \rangle: V \times V \mapsto C$ defined. Prove the famous Cauchy Schwarz inequality which says for all $v, w \in V$

$$|\langle v, w \rangle| \leq \|v\|_2 \|w\|_2$$

3. (*Valentine day special for extra credit:*) Suppose you use a *very* long ribbon to wrap around the whole earth (i. e. let the ribbon tightly circumscribe the whole earth), as though you would give your loved one the whole world nicely wrapped. Now your beloved adds $1m$ to that *very* long ribbon and returns to you (sign of his/her requited love). Now try wrapping the whole earth again with this augmented ribbon. Because it is slightly longer, you can lift the ribbon above the ground uniformly all around the globe. How much can you lift the ribbon off the ground?

Due date: **February 14** in class