Department of Electrical and Computer Engineering
The Johns Hopkins University
520.648 Compressed Sensing and Sparse Recovery – Spring 2017

Homework Assignment II

Reading Assignment: Lecture Notes

Computer Assignment: Sparse Signal Recovery via ℓ_1-minimization.

1. We will mostly repeat what we have done in Assignment I by replacing ℓ_0-minimization with ℓ_1-minimization. We will increase the dimension (you will quickly find out that your existing ℓ_0-minimization strategy will not work anymore). We will also investigate a few more sampling matrices.

   Suppose that we have a signal $\mathbf{x}$ of 256 samples ($N = 256$) where only 5 of these samples are nonzero ($S = 5$). The location and magnitude of these nonzero samples are unknown. Let's investigate the problem of sampling this sparse signal using the various sampling methods.

   Generate the signal using the following Matlab commands:

   $$>> x = zeros(N, 1); q = randperm(N); x(q(1 : S)) = randn(S, 1);$$

   Use Matlab's linear programming function *linprog* to set up the ℓ_1-minimization problem

   $$\hat{\mathbf{x}} = argmin_{\mathbf{x}} \ ||\mathbf{x}||_1 \ \text{ subject to } \ \mathbf{y} = \mathbf{A}\mathbf{x}.$$

   For each sensing matrix $\mathbf{A}$ itemized below, vary the value of M (say $M = \{10, 20, 30, \ldots, 100\}$), and perform ℓ_1-minimization at 100 different instances of the signal $\mathbf{x}$ by varying the location and magnitude of its nonzero samples. Let's say if $||\hat{\mathbf{x}} - \mathbf{x}||_2 \leq 10^{-6}$, then we regard the signal recovery as perfect. Plot the performance curve in which x-axis represents the number of measurements M while y-axis denotes the probability of perfect signal recovery. At each of the 100 instances, the signal should be different and often times, the sensing matrices might be different as well. The following sampling schemes/sensing matrices are under consideration.

   (a) *Random sampling in the time domain*: Suppose $\mathbf{I}$ is the $N \times N$ identity matrix. Create the sensing matrix $\mathbf{A}$ by keeping M rows of $\mathbf{I}$ at random locations (and deleting the remaining $M - N$ rows).

   (b) *Uniform subsampling in the time domain*: The sensing matrix $\mathbf{A}$ in this case is constructed from by selecting M rows of $\mathbf{I}$ whose row indices are in the uniformly-spaced set

   $$\{1, \lfloor N/M \rfloor, 2\lfloor N/M \rfloor, 3\lfloor N/M \rfloor, \ldots, M\lfloor N/M \rfloor\}.$$

(c) *Random sampling in the frequency domain*: Suppose $\mathbf{F}$ is the $N \times N$ DCT matrix ($>> F = dct(eye(N));$). Create the sensing matrix $\mathbf{A}$ by keeping M rows of $\mathbf{F}$ at random locations (and deleting the remaining $M - N$ rows).

(d) *Low-frequency sampling*: Generate the sensing matrix $\mathbf{A}$ by keeping the first M rows of the matrix $\mathbf{F}$.

(e) *Equispaced frequency sampling*: Generate the sensing matrix $\mathbf{A}$ by keeping M rows of the matrix $\mathbf{F}$ at the location

$$\{1, \lfloor N/M \rfloor, 2\lfloor N/M \rfloor, 3\lfloor N/M \rfloor, \ldots, M\lfloor N/M \rfloor\}.$$

(f) *Sampling in a random domain*: The sensing matrix $A \in R^{M \times N}$ in this case is genearted from a collection of random Gaussian variables, then the rows are orthonormalized, i.e.,

$$>> A = randn(M, N); A = orth(A')';$$

Which sensing matrices are best for perfect recovery? In those cases, how many measurements are sufficient for perfect signal recovery ? Which sampling method seems to be most efficient ?

2. Repeat the experiments for the case where the signal is frequency-sparse, i.e., $\mathbf{x}$ only contains $S = 5$ significant frequency components. Like the time-sparse case earlier, the location as well as the magnitude of those S frequencies are unknown. Such a signal can be generated as

$$>> alpha = zeros(N, 1); q = randperm(N); alpha(q(1:S)) = randn(S, 1); x = idct(alpha);$$

How do you modify the ℓ_1-minimization problem for this case? What are your observations this time?

Due date: **Thursday, Feb. 16** in lecture