Department of Electrical and Computer Engineering
The Johns Hopkins University
520.648 Compressed Sensing and Sparse Recovery – Spring 2013

Homework Assignment IV

1. Let $\mathbf{A} = (a_{jk}) \in R^{m \times N}$, $\mathbf{y} \in R^m$, and $\mathbf{x} \in R^N$. Consider the ℓ_0 minimization problem (P_0) as below:

$$(P_0): \quad min_{\mathbf{x} \in R^N} ||\mathbf{x}||_0 \text{ subject to} ||\mathbf{y} - \mathbf{A}\mathbf{x}|| \leq \eta$$

   Show that P_0 is NP-hard by establishing that if one can prove P_0, one can also solve the *Exact- Cover-by-3-Sets Problem*, which is known to be NP-complete. (Hint: follow the steps outlined in class and for now, just assume that $m = 3k$ for some k)

2. Show that $\mu_1(s) = \max_{S \in [N], |S| < s+1} ||\mathbf{A}_S^* \mathbf{A}_S - \mathbf{I}||_{1 \to 1}$. (Hint: Try writing out the matrix $(\mathbf{A}_S^* \mathbf{A}_S - \mathbf{I})$ explicitly and use the definition of the norm of a matrix as a linear operator $|| \cdot ||_{1 \to 1}$, as you did in HW #1).

Due date: **Thursday, March 14** in class