

Homework Assignment IV

Reading Assignment: Lecture Notes

1. Prove the following statement: ℓ_1 -minimization encourages sparse solution.

Let $\mathbf{A} \in \mathcal{R}^{M \times N}$, $\mathbf{y} \in \mathcal{R}^M$, and $\mathbf{x} \in \mathcal{R}^N$. If there is a unique solution \mathbf{x}_1 to the ℓ_1 -minimization problem below

$$(P_1): \min_{\mathbf{x} \in \mathcal{R}^N} \|\mathbf{x}\|_1 \quad \text{subject to } \mathbf{y} = \mathbf{A}\mathbf{x},$$

then $\|\mathbf{x}_1\|_0 \leq M$.

2. With the assumptions in Lemma 2.2 in lecture, show that:

$$|\langle \mathbf{A}\mathbf{u}, \mathbf{A}\mathbf{v} \rangle| = \frac{1}{4} \left| \|\mathbf{A}\mathbf{u} + \mathbf{A}\mathbf{v}\|_2^2 - \|\mathbf{A}\mathbf{u} - \mathbf{A}\mathbf{v}\|_2^2 \right|$$

Due date: **Thurs, Mar. 1** in lecture