Department of Electrical and Computer Engineering
The Johns Hopkins University
520.648 Compressive Sensing and Sparse Recovery – Spring 2013

Homework Assignment V

Computer Assignment:

Consider four reconstruction algorithms: Orthogonal Matching Pursuit (OMP), Subspace Pursuit (SP), Gradient Projection for Sparse Reconstruction (GPSR), and Augmented Lagrange Multipler (ALM). You can find the code package on the course webpage which implements all four algorithms. This exercise helps you to investigate and compare their accuracy as well as robustness in the recovery of sparse signals with various sparsity level s. The sensing matrices in comparison are: Random Gaussian and Random Subsampling. Make sure that you use the same setup for the algorithms to obtain a fair comparison.

- Time-sparse signals: $\mathbf{x}$ of 256 samples ($N = 256$) where only s of these samples are nonzero. The location and magnitude of these nonzero samples are unknown.
- Frequency-sparse signals: $\mathbf{x}$ only contains s significant frequency components. Like the time-sparse case earlier, the location as well as the magnitude of those s frequencies are unknown.

1. Plot the probability of success (exact recovery) of the four algorithms with respect to the sparsity level s and the number of measurements m when the magnitudes of the non-zero entries in $\mathbf{x}$ has a Gaussian (or Laplacian) distribution.
2. Plot the probability of success (exact recovery) of the four algorithms with respect to the sparsity level s and the number of measurements m when the magnitudes of the non-zero entries in $\mathbf{x}$ are exactly (or roughly) the same. Any observation from the two plots above?
3. Repeat the recovery exercise when the signals are three images: Phantom (synthetic), Brain (real) and Boat (real), all available on the course web page in the same code package. This time, devise your own stopping criterion and try to fine-tune other parameter(s) all of the algorithms. The sparsifying matrices that you should consider are: Identity, DCT, and Wavelet. The main.m file in the package helps you to set up the Compressed Sensing problem for these images. Compute the distortion based on the peak signal-to-noise ratio, often abbreviated PSNR, defined as follows

$$PSNR = 10 \log_{10} \frac{MAX^2}{MSE} \quad \text{where} \quad MSE = \frac{1}{N^2} ||\hat{\mathbf{x}} - \mathbf{x}||_2^2.$$

   For our three test images, $MAX = 255$ (the maximum dynamic range) and N is the image dimension. Use the psnr.m file in the package to plot the PSNR between the recovered images and the original with respect to the number of measurements m. Again, what are your observations on how to obtain the best recovery performance?

Due date: **Thursday, March 28** in class