Department of Electrical and Computer Engineering
The Johns Hopkins University
520.648 Compressed Sensing and Sparse Recovery – Spring 2017

Homework Assignment V

Reading Assignment: Lecture Notes and CS proofs by Davenport/Rauhut from our Lectures directory.

1. Implement the FISTA and AMP recovery algorithms. You are encouraged to adapt the GAMP algorithm from the authors' web page. Repeat the exercise in Assignment III for the 2 new algorithms and compare to what you achieved previously. Also, this time, extend the exercise to higher level of s, say $s = 25$ and $s = 50$.

2. Define **mutual coherence** for two orthonormal bases $\mathbf{U} = [\ \mathbf{u_1}, \ldots, \mathbf{u_N}\ ]$ and $\mathbf{V} = [\ \mathbf{v_1}, \ldots, \mathbf{v_N}\ ]$ as follows
$$\mu(\mathbf{U}, \mathbf{V}) = \max_{1 \le i,j \le N} | < \mathbf{u}_i, \mathbf{v}_j > |.$$
   (a) Establish that
   $$\frac{1}{\sqrt{N}} \le \mu(\mathbf{U}, \mathbf{V}) \le 1$$
   and these inequalities are sharp, in other words, proving that equality exists.
   (b) For any arbitrary non-zero $\mathbf{b} \in \mathcal{R}^N$ such that $\mathbf{b} = \mathbf{U}\boldsymbol{\alpha} = \mathbf{V}\boldsymbol{\beta}$, prove that
   $$||\boldsymbol{\alpha}||_0 + ||\boldsymbol{\beta}||_0 \ge \frac{2}{\mu(\mathbf{U}, \mathbf{V})}.$$

Due date: **March 30** in Lecture