A Family of Lapped Regular Transforms With Integer Coefficients

Masaaki Ikehara, *Senior Member, IEEE*, Trac D. Tran, *Member, IEEE*, and Truong Q. Nguyen, *Senior Member, IEEE*

***Abstract*—Invertible transforms with integer coefficients are highly desirable because of their fast, efficient, VLSI-suitable implementations and their lossless coding capability. In this paper, a large class of lapped regular transforms with integer coefficients (ILT) is presented. Regularity constraints are also taken into account to provide smoother reconstructed signals. In other words, this ILT family can be considered to be an M-band biorthogonal wavelet with integer coefficients. The ILT also possesses a fast and efficient lattice that structurally enforces both linear-phase and exact reconstruction properties. Preliminary image coding experiments show that the ILT yields comparable objective and subjective performance to those of popular state-of-the-art transforms with floating-point coefficients.**

***Index Terms*—Image coding, integer transform, lapped transform.**

I. Introduction

With integer-coefficient transforms, the coefficients representing the signal can be obtained very efficiently by shift-and-add operations, leading to multiplierless systems. Transforms with integer coefficients not only lead to fast, low-powered VLSI implementations but also yield integer output, which is a necessary condition for lossless transform coding. In the two-channel case, there are extensive works in this area, and most solutions have been found (see [4], [5], and references therein). On the other hand, research on integer-coefficient M-channel systems is still at an early stage. It has been shown recently that multiband transforms with floating-point coefficients, when appropriately designed and utilized, can outperform state-of-the-art wavelets by significant margins [6], [7], [9], [11]. The overlapping basis functions of these transforms can eliminate annoying blocking artifacts just as efficiently as any wavelet. However, the wavelet transform requires many more operations per output coefficient, and it may need a large memory buffer in its implementation. LTs are more advantageous than wavelets because they can be implemented as block transforms.

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M. Ikehara is with the Department of Electronics and Electrical Engineering, Keio University, Yokohama, Japan (e-mail: ikehara2tkhm.elec.keio.ac.jp).

T. D. Tran is with the Electrical Engineering Department, The John Hopkins University, Baltimore, MD 21218 USA (e-mail: trac@thanglong.ece.jhu.edu).

T. Q. Nguyen is with the Electrical and Computer Engineering Department, University of California at San Diego, La Jolla, CA 92093 USA (e-mail: nguyen@ece.ucsd.edu).

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The simplest example of an M-channel transform with integer coefficients is the Walsh–Hadamard transform (WHT) [3], whose coefficients consist of either 1 or -1 without normalization factor. The eight-point DWT is defined by

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}.$$

In addition, the discrete WHTs for a size $M = 2^L$ are given nonrecursively. This is an orthogonal matrix. However, the WHT is too simple to achieve good coding performances (analogously to the comparison between the Haar and other high-performance wavelets), and its basis functions are not overlapped. In image coding, the synthesis lowpass filter should be long and smooth to avoid blocking and checkerboarding. To obtain smooth synthesis function, the synthesis lowpass filter is required to have as many vanishing moments as possible. To achieve this, we can utilize the maximally flat Mth-band filter as our synthesis lowpass filter.

In this paper, we present a nontrivial M-channel lapped biorthogonal transforms with integer coefficients, where the synthesis lowpass filter of the WHT is replaced by a factorizable maximally flat Mth-band filter. The replacement increases the transform's efficiency in representing smooth signal components to avoid blocking artifacts. Next, we show that the length of the lowpass filter can be traded off between the analysis and synthesis side by applying balancing [2]. As result, the ILT with arbitrary regularities can be designed. Several lifting steps or a ladder structure can then be applied to improve the transform further. The resulting ILT is biorthogonal, has linear-phase basis functions of variable lengths, and, most importantly, has integer coefficients. The ILT can also be thought of as a class of M-band biorthogonal wavelets with integer coefficients and a fast implementation.

A. Review of Lapped Transform

In this paper, we limit the discussions on lapped transforms to M-channel uniform linear phase perfect reconstruction filterbanks (LPPRFBs). The generalized paraunitary LPPRFBs are called GenLOT and are presented in [8]. The most general lattice for M-channnel linear phase lapped biorthogonal trans-

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forms (GLBTs) is presented in [9] and [10]. The polyphase matrix $\mathbf{E}(z)$ can be factorized as [9]

$$\mathbf{E}(z) = \mathbf{G}_{K-1}\mathbf{G}_{K-2}\ldots\mathbf{G}_1(z)\mathbf{E}_0$$

$$\mathbf{G}_i(z) = \frac{1}{2}\begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_i\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & z^{-1}\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix} \quad (1)$$

$$\triangleq \frac{1}{2}\Phi_i\mathbf{W}\mathbf{\Lambda}(z)\mathbf{W}, \quad (2)$$

and

$$\mathbf{E}_0 = 1\sqrt{2}\begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_0\mathbf{J}_{\frac{M}{2}}\\ \mathbf{V}_0\mathbf{J}_{\frac{M}{2}} & -\mathbf{V}_0^{\frac{}{}}\end{bmatrix}. \quad (3)$$

This lattice results in all filters having length $L = KM$. K is often called the overlapping factor. Each cascading structure $\mathbf{G}_i(z)$ increases the filter length by M. All $\mathbf{U}_i$ and $\mathbf{V}_i, i = 0, 1, \ldots, K-1$ are arbitrary $(M/2) \times (M/2)$ invertible matrices, and they can be completely parameterized by their singular value decomposition (SVD), i.e., $\mathbf{U}_i = \mathbf{U}_{i0}\mathbf{\Gamma}_i\mathbf{U}_{i1}$ and $\mathbf{V}_i = \mathbf{V}_{i0}\mathbf{\Delta}_i\mathbf{V}_{i1}$, where $\mathbf{\Gamma}_i$ and $\mathbf{\Delta}_i$ are diagonal matrices with positive elements.

B. Outline of the Paper

The paper begins with the Mth-band filter. Section II shows the basic ILT structure, using the factorization of the maximally flat Mth-band filter. Section III presents balancing method for M-channel filterbanks, which is well known for the two-channel case. Section IV gives the lifting steps based on the basic ILT to improve the coding performance while keeping the regularities. Finally, Section V investigates the application of the new transform in image coding.

Notations: Boldfaced letters indicate vectors and matrices. Superscript T denotes transposition, and $\mathbf{I}_k$ denotes the $k \times k$ identity matrix.

II. Basic ILT Structure

In this paper, we restrict the class of lapped transforms in the discussion to M-channel uniform linear phase perfect reconstruction filterbanks (LPPRFBs) whose polyphase representation is depicted in Fig. 1 [1]. The neccesary and sufficient condition for perfect reconstruction of the GLBT is expressed by

$$\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}. \quad (4)$$

Now, the analysis and synthesis filters are expressed by $\mathbf{H}(z) = \mathbf{E}(z)\mathbf{Z}(z)$ and $\mathbf{F}^T(z) = z^{-(M-1)}\mathbf{Z}^T(z^{-1})\mathbf{R}(z)$, respectively, where $\mathbf{H}(z) = [H_0(z), H_1(z), \ldots, H_{M-1}(z)]^T$, $\mathbf{F}(z) = [F_0(z), F_1(z), \ldots, F_{M-1}(z)]^T$, and $\mathbf{Z}(z) = [1, z^{-1}, \ldots, z^{-(M-1)}]^T$. In the GenLOT [8], the synthesis filters are time-reversed versions of the analysis filter and $\mathbf{R}(z) = \mathbf{E}^T(z^{-1})$. GenLOT can be constructed by lattice structures that consist of the orthogonal matrices and the diagonal matices with delays. On the other hand, the lattice matrices of the GLBT are not restricted to be orthogonal. With the added degrees of freedom, GLBT outperforms GenLOT in image coding.

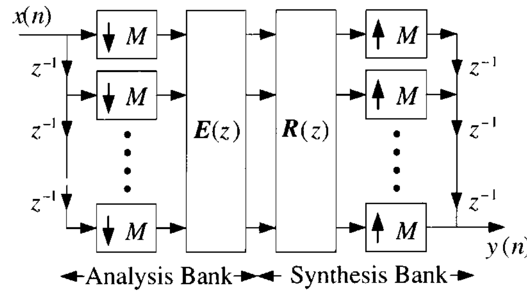

Fig. 1. Polyphase representation of an LPPRFB.

A. Maximally Flat Mth-Band Filters

One way to construct GLBT is to use a spectral factor of the Mth-band filter as the analysis and synthesis filters. The Mth-band filter $P(z)$ satisfies Nyquist's condition, which is defined as $p(Mk) = (\delta(k)/M)$ in the time domain and is expressed as $\sum_{n=0}^{M-1} P(zW^n) = 1$ in the frequency domain. On the other hand, the perfect reconstruction condition (PR) of LPPRFB is expressed by

$$\sum_n h_i(n) f_j(M\ell - n) = \delta(\ell)\delta(i-j) \quad (5)$$

where $h_i(n)$ and $f_j(n)$ are the analysis and synthesis filter, respectively. If the analysis and synthesis filter pair of $h_i(n)$ and $f_i(n)$ are the spectral factors of M-the band filter, one condition of PR is satisfied. Then, the maximally flat Mth-band filter is useful to construct ITL.

Definition: A filter $P(z)$ is said to be a maximally flat Mth-band filter if it has the following form [13]:

$$P(z) = \left[\frac{1 + z^{-1} + \cdots + z^{-(M-1)}}{M}\right]^{2K} Q(z) \quad (6)$$

where

$$Q(z) = \sum_{i=0}^{2K-2} q(i)(z^{-1} - 1)^i$$

$$q(i) = (2K - 1 - i)\sum_{\ell=0}^{2K-2i}\binom{2K-2-i}{\ell} \times (-1)^{i-\ell}B_{\ell+1} \quad i = \{0, 1, \ldots, 2K-2\}$$

and

$$B_\ell = \frac{M}{(2K-1)}\prod_{k=\ell}^{K-1}(\ell^2 - k^2M^2).$$

$P(z)$ and its first $(2K-1)$ derivatives vanish for $z = e^{j2\pi k/M}, k = 1, 2, \ldots, M-1$. When $P(z) = P_0(z)P_0(z^{-1})$, $P_0(z)$ is said to be K-regular M-band unitary scaling filter [13]. Since $Q(z)$ has the binomial and symmetric coefficient, this is clearly a factorizable polynomial with integer coefficients, and $P(z)$ is a polynomial of degree $2(MK-1)$ in z and has linear phase. Thus, the maximally flat Mth-band filter all have binominal coefficients that are integers that are divisible by 2 and have the maximum number of vanishing moments.

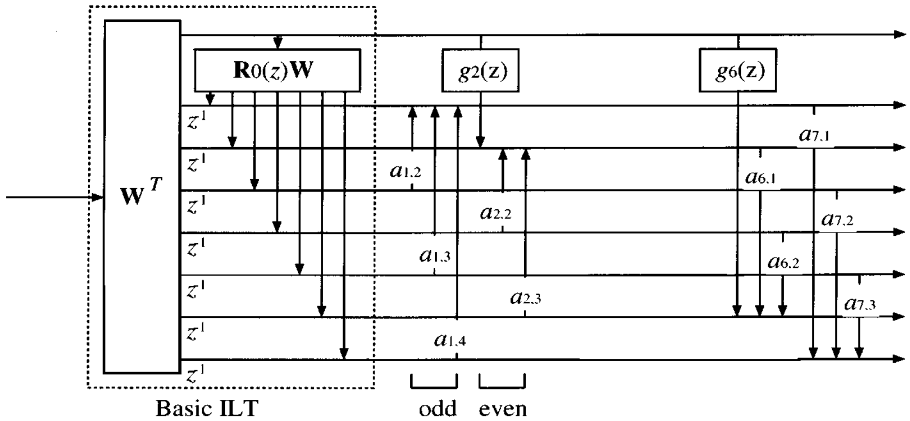

Fig. 2. Analysis bank using lifting scheme.

B. Basic ILT

Since $P(z)$ is an Mth-band filter, its impulse responce $p(n)$ has the following characteristic:

$$p(n) = \begin{cases} \frac{1}{M}, & n = MK - 1 \\ 0, & n = Mp - 1, \quad p\text{: integer}, p \neq K \end{cases} \tag{7}$$

If $P(z)$ is expressed by the polyphase representation, it has the following form:

$$P(z) = \sum_{k=0}^{M-1} z^{-k} P_k(z^M), \quad P_{M-1}(z) = \frac{1}{M} z^{-K} \tag{8}$$

We construct the ILT's polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ from a simple modification of the WHT and a series of lifting steps based on the maximally flat Mth-band filter.

Theorem: Let the synthesis bank be WHT $\mathbf{W}$. If the WHT's synthesis lowpass filters that correspond to the first column of $\mathbf{W}$ are replaced by the following filter $F_0(z)$, which is a factors of $P(z)$

$$\begin{aligned} F_0(z) &= \left[\frac{1 + z^{-1} + \cdots + z^{-(M-1)}}{M}\right]^{2K-1} Q(z) \\ &\triangleq \sum_{i=0}^{M-1} z^{-(M-1-i)} R_{0,i}(z^M) \end{aligned} \tag{9}$$

then the synthesis polyphase matrix $\mathbf{R}(z)$ form a perfect reconstruction FIR system, and the resulting analysis polyphase matrix $\mathbf{E}(z)$ is FIR with linear phase and has binomial coefficients.

Proof: With the proposed synthesis filter $F_0(z)$, the polyphase matrix $\mathbf{R}(z)$ of the synthesis bank now becomes

$$\mathbf{R}(z) = \left[\mathbf{R}_0^T(z) \quad \tilde{\mathbf{W}}\right] \tag{10}$$

where $\mathbf{R}_0(z) = [R_{0,0}(z), R_{0,1}(z), \ldots, R_{0,M-1}(z)]$, and $\tilde{\mathbf{W}}$ is the WHT matrix $\mathbf{W}$ with the first column deleted. To achieve PR in the filterbanks, the analysis lowpass filter $H_0(z)$ is expressed by

$$H_0(z) = \left[\frac{1 + z^{-1} + \cdots + z^{-(M-1)}}{M}\right]. \tag{11}$$

From (9) and (11), we obtain

$$P(z) = H_0(z)F_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{i=0}^{M-1} z^{-(M-1-i+k)} R_{0.i}(z^M). \tag{12}$$

Since $P_{M-1}(z)$ in (8) corresponds to the case $i = k$, $P_{M-1}(z)$ is expressed by

$$P_{M-1}(z) = \sum_{i=0}^{M-1} R_{0,i}(z) = z^{-K}.$$

Therefore, it can be easily shown that

$$[1 \quad 1 \quad \cdots \quad 1]\mathbf{R}(z) = [z^{-K} \quad 0 \quad \cdots \quad 0] \tag{13}$$

where the vector $[1 \ 1 \ \cdots \ 1]$ is the polyphase component of $H_0(z)$ corresponding to the first row of the WHT. Next, we have to prove that the determinant of $\mathbf{R}(z)$ must be a monomial, i.e., the filterbank achieves PR and is FIR.

Multiplying $\mathbf{W}\mathbf{W}^T = \mathbf{I}$ to $\mathbf{R}(z)$ on the left yields

$$\mathbf{W}\mathbf{W}^T\mathbf{R}(z) = \mathbf{W} \begin{bmatrix} z^{-K} & \mathbf{0} \\ \tilde{\mathbf{W}}^T\mathbf{R}_0^T(z) & \mathbf{I}_{M-1} \end{bmatrix}.$$

Hence, $\det(\mathbf{R}(z)) = z^{-K}$. In addition, notice that $F_0(z)$ is factorizable. Therefore, the synthesis bank can be implemented by lifting the submatrix $\tilde{\mathbf{W}}^T\mathbf{R}_0^T(z)$ following the WHT. The corresponding analysis polyphase matrix $\mathbf{E}(z)$ is then given by

$$\mathbf{E}(z) = z^{-K}\mathbf{R}^{-1}(z) = \begin{bmatrix} 1 & \mathbf{0} \\ -\tilde{\mathbf{W}}^T\mathbf{R}_0^T(z) & z^{-K}\mathbf{I}_{M-1} \end{bmatrix} \mathbf{W}^T.$$

The analysis bank is also implementable using lifting, as illustrated in Fig. 2. It is noted that the lifting filters $\tilde{\mathbf{W}}^T\mathbf{R}_0^T(z)$ have integer coefficients. We label the combination of $\mathbf{E}(z)$ and $\mathbf{R}(z)$ above as the basic ILT. □

The basic ILT is constructed by WHT and a series of lifting steps, as shown in Fig. 2. As result, the analysis lowpass filter has length M, and other filters have length $M(2K-1)$. Inversely, the synthesis lowpass filter has length $M(2K-1)$ and is $(2K-1)$ regular. It is noted that the total length of the analysis and corresponding synthesis filters is $2MK$.

III. Balancing

The lowpass analysis filter in the basic ILT is one-regular, and the lowpass synthesis filter is $(2K-1)$-regular. However, changing the filter length and regularities are required in some applications [2]. Since the length of the analysis lowpass filter is very short compared with the synthesis lowpass filter, one wants to trade the length between the analysis and synthesis side while keeping the integer coefficients and PR. Then, some of the factor $((1+z^{-1}+\cdots+z^{-(M-1)})/M)$ in the synthesis bank should be moved to the analysis bank while keeping some regularities. This operation is called balancing, and it is easy to move $((1+z^{-1})/2)$ in the two-channel filterbank [2]. We extend balancing to M-channel basic ILT.

Moving $((1+z^{-1}+\cdots+z^{-(M-1)})/M)$ from $F_0(z)$ to $H_0(z)$ maintains integer coefficients and symmetry.

$$\begin{aligned} h_0^{\text{new}}(n) &= \frac{1}{M}\sum_{k=0}^{M-1} h_0(n-k) \\ f_0^{\text{new}}(n) &= \frac{1}{M}\left[f_0(n) - \sum_{k=1}^{M-1} f_0^{\text{new}}(n-k)\right] \end{aligned} \tag{14}$$

The product $H_0^{\text{new}}(z)F_0^{\text{new}}(z)$ equals $H_0(z)F_0(z)$ and the maximally flat Mth-band filter $P(z)$. However, this operation destroys the perfect reconstruction in the filterbank. Then, we apply this operation to all analysis and synthesis filters.

Multiplying $H_i(z)$ by $((1+z^{-1}+\cdots+z^{-(M-1)})/M)$ is expressed in the polyphase representation

$$\begin{aligned} \hat{\mathbf{E}}^{\text{new}}(z) &= \frac{1}{M}\mathbf{E}(z)\begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ z^{-1} & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ z^{-1} & \cdots & z^{-1} & 1 & 1 \\ z^{-1} & \cdots & \cdots & z^{-1} & 1 \end{bmatrix} \\ &= \mathbf{E}(z)\mathbf{\Gamma}(z). \end{aligned} \tag{15}$$

Dividing $F_i(z)$ by $((1+z^{-1}+\cdots+z^{-(M-1)})/M)$ is expressed in the polyphase representation

$$\begin{aligned} \hat{\mathbf{R}}^{\text{new}}(z) &= \frac{M}{1-z^{-1}}\begin{bmatrix} 1 & -1 & 0\cdots & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \\ -z^{-1} & 0 & \cdots & 0 & 1 \end{bmatrix}\mathbf{R}(z) \\ &= \mathbf{\Gamma}^{-1}(z)\mathbf{R}(z). \end{aligned} \tag{16}$$

Because the determinant of $\mathbf{\Gamma}(z)$ is $((1-z^{-1})/M)^{(M-1)}$, $\mathbf{\Gamma}^{-1}(z)$ is the rational matrix polynomial. If we use this form as it is, the synthesis filters are not FIR but IIR. Then, the polyphase matrices are transformed such that the determinant of the new polyphase matrix is a monomial, and the lowpass filter is invariable.

$$\begin{aligned} \mathbf{E}^{\text{new}}(z) &= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{1-z^{-1}} & 0\cdots & \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{1}{1-z^{-1}} \end{bmatrix}\mathbf{E}(z)\mathbf{\Gamma}(z) \\ \mathbf{R}^{\text{new}}(z) &= \mathbf{\Gamma}^{-1}(z)\mathbf{R}(z) \\ &\quad\times\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & (1-z^{-1}) & 0 & \cdots \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & (1-z^{-1}) \end{bmatrix}. \end{aligned} \tag{17}$$

It is noted that $\mathbf{E}^{\text{new}}(z)$ and $\mathbf{R}^{\text{new}}(z)$ still have PR. As a result, the synthesis filters are expressed by

$$\begin{aligned} F_0^{\text{new}}(z) &= \left[\frac{1+z^{-1}+\cdots+z^{-(M-1)}}{M}\right]^{2K-2} Q(z) \\ F_i^{\text{new}}(z) &= M(1-z^{-1})F_i(z) \quad \text{for } i=1,2,\ldots,M-1. \end{aligned} \tag{18}$$

Thus all synthesis filters have binomial coefficients. Similar to the synthesis bank, the analysis filters are expressed by

$$\begin{aligned} H_0^{\text{new}}(z) &= \left[\frac{1+z^{-1}+\cdots+z^{-(M-1)}}{M}\right]^2 \\ H_i^{\text{new}}(z) &= \left[\frac{1+z^{-1}+\cdots+z^{-(M-1)}}{M(1-z^{-M})}\right]H_i(z) \\ &= \frac{1}{M(1-z^{-1})}H_i(z) \quad \text{for } i=1,2,\ldots,M-1. \end{aligned} \tag{19}$$

Thus, $H_i^{\text{new}}(z)$ is the rational function. Since the frequency response of the scaling filter corresponding to $F_0(z)$ has a zero order $2K-1$ at the Mth roots of unity, all moments up to order $2K-2$ of the wavelet corresponding to $H_i(z)$ vanish [13]. That is, $H_i(z)$ has $2K-1$ zeros at $z=1$. Therefore, the denominator of $H_i^{\text{new}}(z)$ is cancelled by the numerator, and $H_i^{\text{new}}(z)$ is the FIR transfer function and still has binomial coefficients. By recursively applying this algorithm $2K-1$ times, we can obtain $2K$ kinds of filterbanks with integer coefficients. When this balancing algorithm is applied s times, which means $H_0^{\text{new}}(z)=((1+z^{-1}+\cdots+z^{-(M-1)})/M)^{s+1}$, the resulting analysis lowpass filter has length $M(s+1)-s$, and the other analysis filter has length $M(2K-1)-s$. The total length of the analysis and corresponding synthesis filters is still $2MK$. Similarly to the two-channel case, the balancing operation may make the frequency response poor; however, we can show the balancing for M-channel, which has never known.

Example 1: A basic eight-channel ILT with $K=2$ in (9) is designed. The analysis lowpass filter has length 8, and other bandpass filters have length 24. In the synthesis bank, the lowpass filter has length 24, and the rest have length 8. Then, a $((1+z^{-1}+\cdots+z^{-(M-1)})/M)$ is moved from $F_0(z)$ to $H_0(z)$. In the resulting ILT, the analysis lowpass filter has length 15, and other bandpass filters have length 23. In the synthesis bank of the resulting ILT, the lowpass filter has length 17, and the rest have length 9. It is noted that the total length of the resulting ILT is unchanged even if the balancing is applied. Fig. 3(a) and (b) shows the magnitude response of the resulting balanced ILT.

IV. Further Lifting Steps

A. *Lifting*

When we consider image coding application, the resulting ILT is not enough to achieve good coding performances because the frequency response is poor. Then, several lifting steps or ladder structure are applied to the basic ILT to improve the transform further. Fig. 4(a) and (b) show the magnitude and impulse responses of the analysis and synthesis filters in the eight-channel basic ILT with $K=2$. All filters have integer

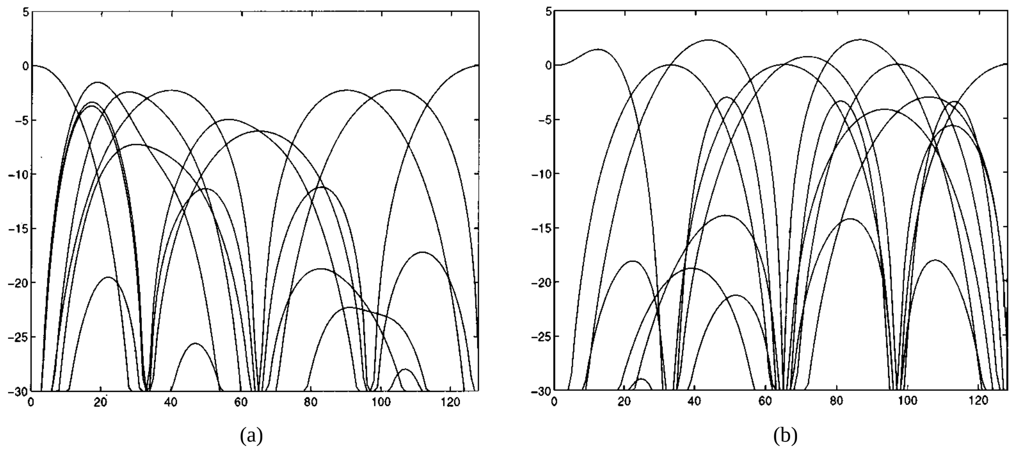

Fig. 3. Magnitude responses of (a) the balanced ILTs analysis bank. (b) Balanced ILTs synthesis bank.

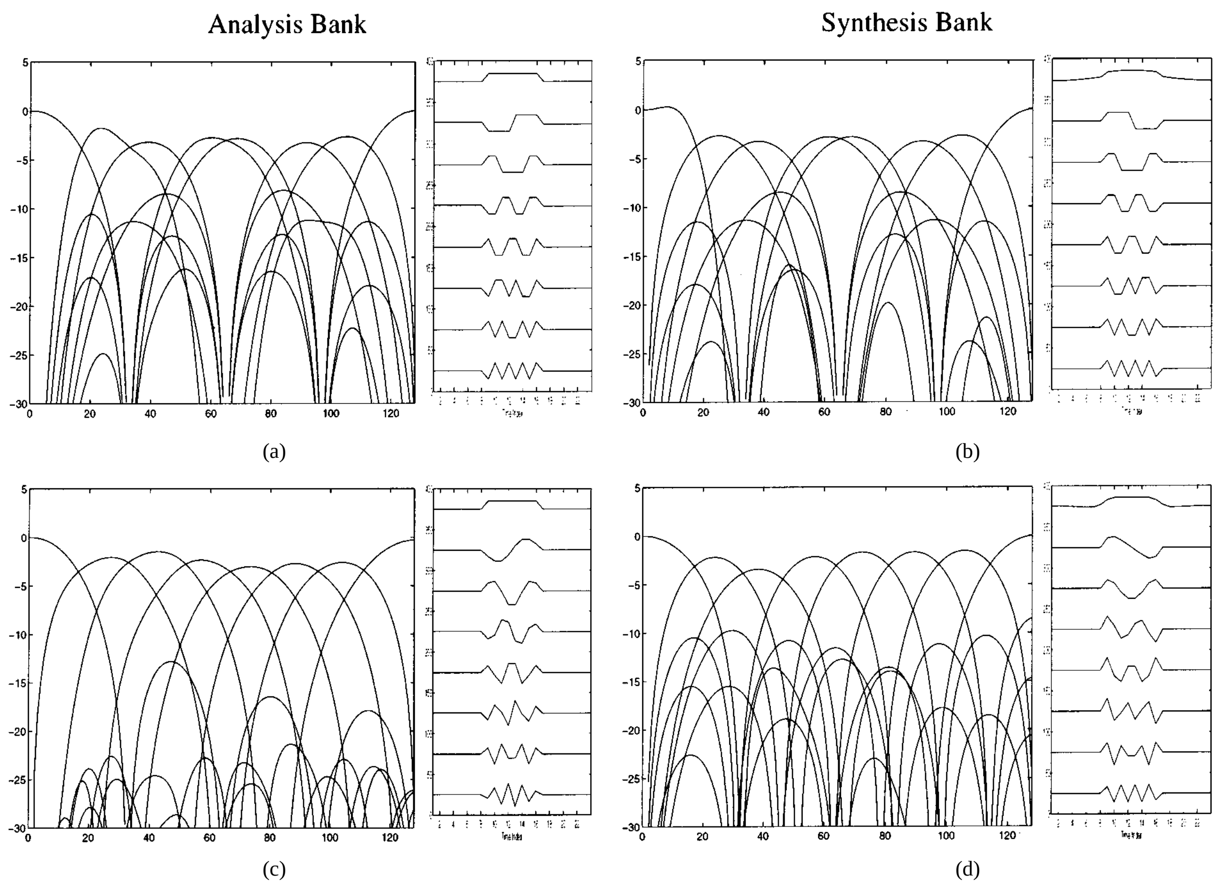

Fig. 4. Magnitude and impulse responses of (a) basic ILTs analysis bank, (b) basic ILT's synthesis bank, (c) lifted ILTs analysis bank, and (d) lifted ILTs synthesis bank.

coefficients so far. Since the synthesis lowpass filter has length 24 and three-regularities, this can eliminate the blocking artifacts. Since the other bandpass filters have length 8, these filters can also avoid the ringing artifacts. Therefore, a basic ILT is suitable for image compression. However, the analysis bank does not have high enough attenuation and coding gain. We can improve our basic ILT further by applying the following lifting steps while keeping the filter length and symmetric polarity.

1) For $i = 1 : M/2$.
2) Since the analysis filters $H_{2i-1}(z)$ with odd indices are antisymmetric and have the same length $M(2K-1)$, each analysis filter with odd index is lifted by only scaled odd index ones to keep antisymmetry and length.

$$H_{2i-1}(z) = H_{2i-1}(z) + \sum_{j=1, j\neq i}^{M/2} a_{2i-1,j} H_{2j-1}(z). \qquad (20)$$

In the synthesis bank, each synthesis filter is lifted inversely:

$$F_{2j-1}(z) = F_{2j-1}(z) - a_{2i-1,j} F_{2i-1}(z) \quad \text{for } j = 1, 2, \ldots, M/2.$$

TABLE I
COMPARISON OF TRANSFORM COMPLEXITY: NUMBER OF OPERATIONS PER 8 TRANSFORM COEFFICIENTS

Transform	*No.of Multiplications*	*No. of Additions*	*Total No. of Ops.*
9/7 Wavelet, 3-level	63	98	161
8×8 DCT	13	29	42
8×16 GLBT	64	96	160
LILT	31	55	86

TABLE II
OBJECTIVE CODING RESULT (PSNR IN DECIBELS) USING DIFFERENT TRANSFORMS ON TEST IMAGES LENA, GOLDHILL, AND BARBARA

	Lena				Goldhill				Barbara			
Comp Ratio	SPIHT	8×8 DCT	8×16 GLBT	8×24 LILT	SPIHT	8×8 DCT	8×16 GLBT	8×24 LILT	SPIHT	8×8 DCT	8×16 GLBT	8×24 LILT
1:8	40.41	39.91	40.35	40.08	36.55	36.25	36.69	36.41	36.41	36.31	37.84	36.63
1:16	37.21	36.38	37.28	36.82	33.13	32.76	33.31	32.99	31.40	31.11	33.02	31.66
1:32	34.11	32.90	34.14	33.63	30.56	30.07	30.70	30.43	27.58	27.28	29.04	27.93
1:64	31.10	29.67	31.04	30.63	28.48	27.93	28.58	28.41	24.86	24.58	26.00	25.24

3) Since the analysis filters $H_{2i}(z)$ with even indices are symmetric, each analysis filter, except for the lowpass filter, is lifted by only even index ones. Since the analysis lowpass filter $H_0(z)$ has length M and other analysis filters $H_{2i}(z)$ has length $M(2K-1)$, the difference between $H_0(z)$ and $H_{2i}(z)$ is $2M(K-1)$. Therefore, $H_0(z)$ is lifted after multiplying the symmetry polynomial $g_{2i}(z)$ with order $2(K-1)$ to make the length same and keep symmetry.

$$
\begin{aligned}
&H_{2i}(z) = H_{2i}(z) + g_{2i}(z^M)H_0(z) + \sum_{j=1, j\neq i}^{M/2-1} a_{2i,j}H_{2j}(z)\\
&\text{where } g_{2i}(z) = a_{2i,0}(1-z^{-1})^{2(K-1)}\\
&F_0(z) = F_0(z) - g_{2i}(z^M)F_{2i}(z)\\
&F_{2j}(z) = F_{2j}(z) - a_{2i,j}F_{2i}(z) \;\; \text{for } j = 1, 2, \ldots, M/2.
\end{aligned}
\tag{21}
$$

Since $F_0(z)$ is length $M(2K-1)$ and $F_{2i}(z)$ is length M, the length of $F_0(z)$ is kept.

4) end

Note that the analysis filters with even and odd indices are lifted by only the even and odd filter, respectively. Fig. 2 shows the analysis bank using the lifting scheme. One can select arbitrarily symmetric polynomials g_{2i} with order $2(K-1)$. However, the synthesis lowpass filter $F_0(z)$ should be kept long and have some degrees of regularity to avoid blocking and checkerbord distortion. Since the synthesis lowpass filter of the basic ILT is $2K-1$ regular, and $g_{2i}(z)$ are $2(K-1)$-regular as well, the resulting synthesis lowpass filter is guanranteed to be at least $2(K-1)$-regular. If one does not need higher regularities, one can select $g_{2i}(z)$ as arbitrarily symmetric polynomials.

B. Optimization

The lifted ILT can be designed such that the bandpass and highpass filters in the analysis bank have enough stopband attenuation in the low-frequency region and the coding gain of the transform is maximized since we are interested mainly in image coding applications. There are $(M-1)(M/2-1)$ free parameters for lifting. The cost function used in this paper is a weighted linear combination of coding gain and stopband attenuation.

$$\Phi = \alpha_1 C_{\text{codinggain}} + \alpha_2 C_{\text{stopband}}. \tag{22}$$

Generally, dc leakage and attenuation around mirror frequencies are added to the cost function for image coding application [12]. However, we do not take care of these cost functins because ILT already has some reguralities. The set of $\{\alpha_i\}$ controls the tradeoff between various filter bank characteristics. We found that the set $\{10,\ 1\}$ works well for image coding. The resulting lifting coefficients are rounded to become binary. This may influence the optimized frequency response and the coding gain. However, we can ignore these effects since the basic ILT already has good responses. Fig. 4(c) and (d) shows the magnitude and impulse responses of the lifted analysis and synthesis filters with binary lifting coefficients.

C. Comparison

The comparison of computational complexity between the ILT and other popular transforms are tabulated in Table I. It is noted that the lifted ILT(LILT) is implemented by shift-and-add operations because all multiplications in ILT are binary. Thus, LILT is faster than various popular transforms in spite of the fact that the coding performance is comparable.

V. APPLICATION IN IMAGE CODING

The coding performance of the new ILT is evaluated through an image coding comparison. To be fair, the same transform-

(a) (b) (c) (d)

Fig. 5. Coding results of Barbara at 1:32 compression ratio. (a) SPIHT, $9/7$-tap biorthogonal wavelet. (b) Embedded 8×8 DCT. (c) Embedded 8×16 GLBT. (d) Embedded 8×24 LILT.

based progressive image coder SPIHT [14] is used in all cases. The difference lies at the transform stage, where the transforms in comparison are the following:

- $9/7$-tap biorthogonal wavelet;
- DCT, eight filters, all 8-tap;
- GLBT, eight filters, all 16-tap [9];
- lifted ILT(LILT), eight filters, only lowpass filter has 8-tap, others have 24-tap.

It is noted that the total filter length of the lifted ILT are the same as that of GLBT. In the latter three uniform-band block-transform cases, we used a modified zerotree structure, where the transform coefficients are grouped in wavelet-like quad trees, and the dc band can have several levels of wavelet decomposition, depending on the image size. For more details on the embedded coding algorithms, see [7], [11], and [14].

Compared with other popular transforms, our lifted ILT performs well on all test images (Lena, Barbara, and Goldhill), as shown by the objective coding results in Table II and the reconstructed images in Fig. 5(a)–(d). Despite having only one long filter of 24 taps, the lifted ILT proves to be very efficient in eliminating blocking artifacts. Ringing is also minimal.

VI. Conclusion

This paper introduces a class of lapped biorthogonal transforms with integer coefficients and variable-length basis functions known as ILT. The ILT is built on a simple modification of the WHT and a series of lifting steps. Hence, it is fast computable via only shift-and-add operations. Image coding examples show that the new integer-coefficient transform consistently yields comparable coding performance with those of state-of-the-art transforms with much higher complexity.

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Masaaki Ikehara (SM’01) received the B.E., M.E., and Ph.D. degrees in electrical engineering from Keio University, Yokohama, Japan, in 1984, 1986, and 1989, respectively.

He was with the Department of Electrical and Computer Engineering, Nagasaki University, Nagasaki, Japan, as an Assistant Professor from 1989 to 1992. In 1992, he joined the Department of Electrical Engineering, Keio University. From 1996 to 1998, he was a Visiting Researcher at the University of Wisconsin, Madison, and Boston University, Boston, MA. He is currently an Associate Professor with the Department of Electronics and Electrical Engineering, Keio University. His research interests are in the areas of multirate signal processing, wavelet image coding, and filter design problems.

Prof. Ikehara has served as Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING since 2001.

Trac D. Tran (M’98) received the B.S. and M.S. degrees from the Massachusetts Institute of Technology, Cambridge, in 1994 and the Ph.D. degree from the University of Wisconsin, Madison, in 1998, all in electrical engineering.

He joined the Department of Electrical and Computer Engineering, The John Hopkins University, Baltimore, MD, in July 1998 as an Assistant Professor. His current research interests are in the field of digital signal processing, particularly in multirate systems, filterbanks, wavelets transforms, and their applications in signal representation, compression, processing, and communications.

Dr. Tran was the Co-Director of the 33rd Annual Conference on Information Sciences and Systems (CISS’99), Baltimore, in March 1999. He received the NFS CAREER award in 2001.

Truong Q. Nguyen (SM’95) received the B.S., M.S., and Ph.D. degrees in electrical engineering from the California Institute of Technology (Caltech), Pasadena, in 1985, 1986, and 1989, respectively.

He was with Lincoln Laboratory, Massachusetts Institute of Technology (MIT), Lexington, from June 1989 to July 1994, as a Member of Technical Staff. From 1993 to 1994, he was a Visiting Lecturer at MIT and an Adjunct Professor at Northeastern University, Boston, MA. From August 1994 to July 1998, he was with the Electrical and Computer Engineering Department, University of Wisconsin, Madison. He was with Boston University, Boston, MA, from 1996 to 2001. He is now a Full Professor at University of California at San Diego, La Jolla. His research interests are in the theory of wavelets and filterbanks and applications in image and video compression, telecommunications, bioinformatics, medical imaging and enhancement, and analog/digital conversion. He is the coauthor (with Prof. G. Strang) of a popular textbook, *Wavelets and Filter Banks* (Welleley, MA: Wellesley-Cambridge, 1997), and the author of several MATLAB-based toolboxes on image compression, electrocardiogram compression, and filterbank design. He also holds a patent on an efficient design method for wavelets and filterbanks and several patents on wavelet applications, including compression and signal analysis.

Prof. Nguyen received the IEEE TRANSACTIONS ON SIGNAL PROCESSING Paper Award (in the Image and Multidimensional Processing area) for the paper he co-wrote with Prof. P. P. Vaidyanathan on linear-phase perfect-reconstruction filter banks in 1992. He received the NSF Career Award in 1995 and is currently the Series Editor (Digital Signal Processing) for Academic Press. He served as Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1994 to 1996 and for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS from 1996 to 1997. He received the Technology Award from Boston University for his invention on Integer DCT (with Y. J. Chen and S. Oraintara). He co-organizes (with Prof. G. Strang at MIT) several workshops on wavelets and served on the DSP Technical Committee for the IEEE CAS Society.