On M-Channel Linear Phase FIR Filter Banks and Application in Image Compression

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***Abstract*—This paper investigates the theory and structure of a large subclass of M-channel linear-phase perfect-reconstruction FIR filter banks—systems with analysis and synthesis filters of length $L_i = K_i M + \beta$, where β is an arbitrary interger, $0 \leq \beta < M$, and K_i is any positive interger. For this subclass of systems, we first investigate the necessary conditions for the existence of linear-phase perfect-reconstruction filter banks (LPPRFB's). Next, we develop a complete and minimal factorization for all even-channel linear-phase paraunitary systems (the most general lapped orthogonal transforms to date). Finally, several design examples as well as comparisons with previous generalized lapped orthogonal transforms (GenLOT's) in image compression are presented to confirm the validity of the theory.**

I. Introduction

In various speech, image, and communications applications, digital filter banks have been used extensively. In this paper, we consider the maximally decimated M-channel filter banks as shown in Fig. 1. In the analysis stage, the signal $x(n)$ is passed through a bank of M analysis filters $H_i(z)$, each of which preserves a frequency band, and the M output signals are then decimated by M to preserve the system's overall sampling rate (thus justifying its name—maximally decimated filter bank). The resulting M subband signals can be treated (coded, processed, and/or transmitted) independently. In the synthesis stage, the M subband signals are combined by interpolators and a set of M synthesis filters $F_i(z)$ to form the reconstructed signal $y(n)$. Filter banks that yield the output $y(n)$ as a time-delayed version of the input $x(n)$, i.e., $y(n) = cx(n - n_0)$, $c \neq 0$, are called perfect reconstruction filter banks (PRFB's).

In some applications, i.e., image processing, it is very crucial for all analysis as well as synthesis filters to have linear phase (either symmetric or antisymmetric). Additionally, linear-phase filters allow us to use simple symmetric extension methods to accurately handle finite-length signals' boundaries. From this point on, all of the filter banks in discussion are linear-phase perfect-reconstruction filter banks (LPPRFB's). In addition, for practical purposes, we only consider real, causal, FIR systems.

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The M-channel filter bank in Fig. 1 can also be represented in terms of its polyphase matrices as shown in Fig. 2(a), where $\mathbf{E}(z)$ is the polyphase matrix corresponding to the analysis filters, and $\mathbf{R}(z)$ is the polyphase matrix of the synthesis filters. Using the noble identities in multirate signal processing [6], one can verify that the filter bank in Fig. 2(b) is another equivalent form. Certainly, if $\mathbf{E}(z)$ is invertible with minimum-phase determinant (for stable inverse), one can obtain a PRFB by choosing $\mathbf{R}(z) = \mathbf{E}^{-1}(z)$. In this paper, *perfect reconstruction* implies that the polyphase matrix $\mathbf{E}(z)$ is invertible. More specifically speaking, any choice of $\mathbf{R}(z)$ that satisfies

$$\mathbf{R}(z)\mathbf{E}(z) = bz^{-l}\mathbf{I}, \quad b \neq 0,\ l \geq 0 \tag{1}$$

also yields perfect reconstruction. We call such systems *biorthogonal*. An effective choice for $\mathbf{E}(z)$ is a paraunitary matrix with

$$\tilde{\mathbf{E}}(z)\mathbf{E}(z) = z^{-l}\mathbf{I}, \quad l \geq 0 \tag{2}$$

where $\tilde{\mathbf{E}}(z) = z^{-K}\mathbf{E}^{\dagger}(z^{-1})$, and K is the order of $\mathbf{E}(z)$. *Paraunitary* systems are guaranteed to satisfy the perfect reconstruction condition with $\mathbf{R}(z)$ chosen to be $\tilde{\mathbf{E}}(z)$.

A. Review of Previous Works

Many works have explored the theory, structures, and design methods of linear-phase FIR perfect-reconstruction filter banks. Most deals with two-channel systems [6], [17], [19], [21], and all solutions have been found. The type A system ($\beta = 0$) has even-length filters with different symmetry polarity (one symmetric and the other antisymmetric). The type B system ($\beta = 1$) has odd-length filters with the same symmetry polarity (both symmetric). Complete and minimal lattice structures for both systems were reported in [17]. However, for M-channel cases, there are still many open problems. First of all, it is not clear what the permissible choices of filters' symmetry polarity and lengths are. This issue has been studied by a number of authors [1], [3], [24], but their results are either not tight enough or not general enough. On design methods, many approaches have been considered, but none has been able to cover the complete set of solutions. Saghizadeh and Willson [22] presented a design method based on optimizing the impulse responses of the analysis filters directly. Another interesting approach was presented by Basu

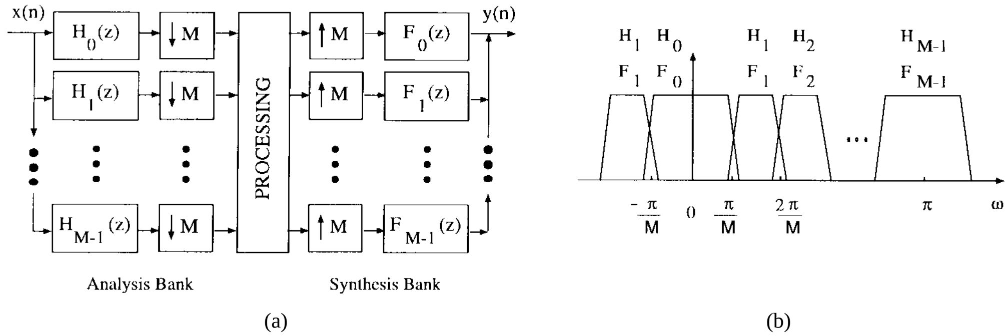


Fig. 1. Maximally decimated M-channel filter bank.

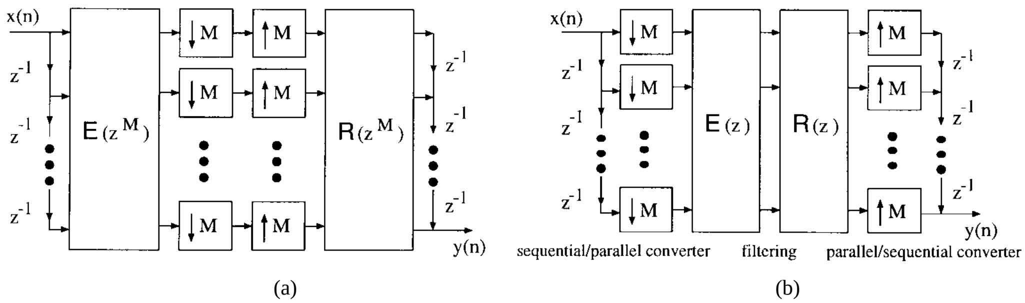


Fig. 2. Equivalent polyphase representations of an M-channel filter bank.

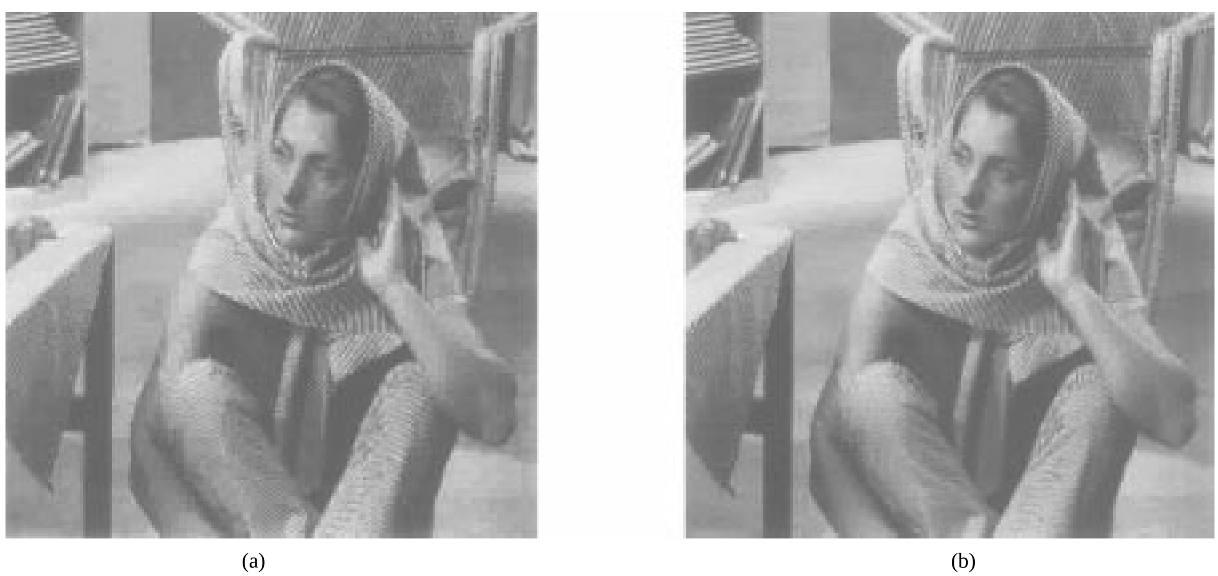


Fig. 3. Coded image at 0.25 b/pixel using (a) DCT and (b) eight-channel length 32 GenLOT.

et al. [24]. This approach presents a complete parametrization of multiband linear-phase biorthogonal filter banks based on the Hermite reduction method of linear system theory on each individual filter. The resulting structure is not minimal, however. Moreover, both linear-phase and perfect-reconstruction properties are not structurally enforced, i.e., these properties will suffer from coefficient quantization.

Lattice structure for the M-channel linear-phase paraunitary filter bank is first presented in [1] and is proven to be both complete and minimal for even-channel and $\beta = 0$ cases. An alternate form of this factorization is given in [2] with the name generalized lapped orthogonal transform (GenLOT), of which the linear phase lapped orthogonal transform (LOT) [11] is a special case. These two designs are attractive because their minimality nature leads to effective optimization procedures, and their completeness guarantees that no optimal solution will be missed using these design methods. These filter banks are found to be very useful in transform-based coding of images [25]–[27]. A typical example in Fig. 3 shows that an eight-channel length-32 linear phase paraunitary system used in a transform-based coder with overlapped input can elegantly eliminate blocking artifacts in the reconstructed image at a rather low bit rate (0.25 b/pixel). Nevertheless, GenLOT design can only give filters with length KM, where M is the number of channels. They are certainly not as general as claimed.

B. Outline of the Paper

Taking a step toward unifying the field of linear-phase M-channel perfect-reconstruction filter bank design, we derive several necessary restrictions on the filters lengths and symmetry polarity in Section II. For practical purposes, only systems with filter lengths $K_iM+\beta,\ i=0,1,\ldots,M-1$ are considered since it has been proven that all β_i have to be the same if one needs to use symmetric extension in the filter bank's implementation [4]. These restrictions are crucial for the convergence of optimization processes for time-domain approaches. They also help in the derivation of a complete factorization for all even-channel linear-phase paraunitary filter banks. The resulting lattice structure is based on orthogonal matrices and delay elements. The number of delay elements used is proven to be minimal. Several design examples obtained from the new lattice structure are presented. They are compared to GenLOT (special case when $\beta=0$) in coding gain, stopband attenuation, and attenuation at dc and around mirror frequencies. A simple image coding example is also provided.

C. Notations

Bold-faced characters are used to denote vectors and matrices. $\mathbf{A}^\dagger$, $\mathbf{A}^T$, $\mathbf{A}^{-1}$, tr($\mathbf{A}$), and $|\mathbf{A}|$ denote, respectively, the conjugate transpose, the transpose, the inverse, the trace, and the determinant of the matrix $\mathbf{A}$. Special matrices used extensively are the identity matrix $\mathbf{I}$, the reversal matrix $\mathbf{J}$, the null matrix $\mathbf{0}$, and the diagonal matrix $\mathbf{D}$ with entries being $+1$ or -1. When the size of a matrix is not clear from context, subscripts will be included to indicate its size. For example, $\mathbf{J}_M$ denotes the $M\times M$ reversal matrix, and $\mathbf{0}_{M\times N}$ denotes the $M\times N$ null matrix. Superscript asterisk, as in $h^*(n)$, denotes conjugation of the coefficients of $h(n)$. For abbreviations, we use LP, PR, PU, and FB to denote, respectively, *linear phase*, *perfect reconstruction*, *paraunitary*, and *filter banks*. *Symmetric* and *antisymmetric* are sometimes abbreviated as S and A, respectively.

II. NECESSARY CONDITIONS FOR LINEAR-PHASE PERFECT-RECONSTRUCTION FILTER BANKS

Consider an M-channel FB with a set of LP analysis filters $H_i(z),\ i=0,1,\ldots,M-1$. Let the length of the ith filter $H_i(z)$ be $L_i=K_iM+\beta$, where $\beta, K_i\in Z_+$, and $0\le\beta<M,\ K_i\ge 1$. Since K_i is arbitrary, the filter's length L_i is also arbitrary. However, the same β is required for all $H_i(z)$ because of the practical usage of symmetric extension in implementation as previously mentioned [4]. For the polyphase matrix $\mathbf{E}(z)$ of this set of filters, the LP property implies that

$$\mathbf{E}(z)=\mathbf{D}\hat{\mathbf{Z}}(z)\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z),\quad\text{where}\quad \hat{\mathbf{J}}(z)=\begin{bmatrix} z^{-1}\mathbf{J}_\beta & \mathbf{0}_{\beta\times(M-\beta)}\\ \mathbf{0}_{(M-\beta)\times\beta} & \mathbf{J}_{M-\beta}\end{bmatrix} \tag{3}$$

where $\hat{\mathbf{Z}}(z)$ is the diagonal matrix diag$[z^{-(K_0-1)}\ \ z^{-(K_1-1)}\ \cdots\ z^{-(K_{M-1}-1)}]$, and $\mathbf{D}$ is a diagonal matrix whose entry is $+1$ when the corresponding filter is symmetric and -1 when the corresponding filter is antisymmetric. $\hat{\mathbf{Z}}(z)$ takes care of the arbitrary lengths of the filters (with different K_i), whereas $\hat{\mathbf{J}}(z)$ accounts for the fact that the first β polyphase components of each filter are one order higher than the rest.

In order to see why $\mathbf{E}(z)$ satisfies (3), let us examine a LP filter $h_i(n)$ with length $L=K_iM+\beta$ and its corresponding polyphase components $\mathbf{E}_{i0}(z),\mathbf{E}_{i1}(z),\ldots,\mathbf{E}_{i(M-1)}(z)$. Since the filter length is a multiple of M plus β, the first β polyphases have one more coefficient than the others

$$\begin{cases}\mathbf{E}_{i\ell}(z)=\sum_{k=0}^{K_i}h_i(kM+\ell)z^{-k}, & \text{for } 0\le\ell\le\beta-1\\ \mathbf{E}_{i\ell}(z)=\sum_{k=0}^{K_i-1}h_i(kM+\ell)z^{-k}, & \text{for } \beta\le\ell\le M-1.\end{cases} \tag{4}$$

First, let us examine $\mathbf{E}_{i0}(z)$ and $\mathbf{E}_{i(\beta-1)}(z)$ and show that they are time-reversed versions of each other

$$\begin{aligned}\tilde{\mathbf{E}}_{i(\beta-1)}(z)&\\ &\triangleq z^{-K_i}\mathbf{E}_{i(\beta-1)}(z^{-1})\\ &=h_i(K_iM+\beta-1)+h_i(K_iM+\beta-1-M)z^{-1}\\ &\quad+\cdots+h_i(\beta-1)z^{-K}\\ &=\pm\sum_{k=0}^{K_i}h_i(kM)z^{-k}=\pm\mathbf{E}_{i0}(z)\end{aligned} \tag{5}$$

where we have used the fact that $h_i(n)$ is LP: $h_i(n)=\pm h_i(K_iM-\beta-1-n)$. Similarly, $\tilde{\mathbf{E}}_{i(M-1)}(z)=\pm\mathbf{E}_{i\beta}(z)$. The $\pm$ sign is used to denote the two separate cases: $h_i(n)$ is symmetric or antisymmetric. This LP property in (5) can be generalized to the rest of the polyphase components to obtain

$$\begin{cases}\mathbf{E}_{i\ell}(z)=\pm\tilde{\mathbf{E}}_{i(\beta-1-\ell)}(z), & \text{for } 0\le\ell\le\beta-1\\ \mathbf{E}_{i\ell}(z)=\pm\tilde{\mathbf{E}}_{i(M+\beta-1-\ell)}(z), & \text{for } \beta\le\ell\le M-1\end{cases}. \tag{6}$$

Using (6), one can easily verify the property of $\mathbf{E}(z)$ in (3).

It should be noted that (3) is an extension of the LP constraint proposed in [21] and is later used extensively in [1]. In fact, if all of our filters are of the same length $L=KM$ (the special case when $K_i=K$ and $\beta=0$), (3) reduces to $\mathbf{E}(z)=\mathbf{D}z^{-(K-1)}\mathbf{E}(z^{-1})\mathbf{J}_M$.

Using this form of $\mathbf{E}(z)$ in (3), the trace and the determinant of $\mathbf{D}$ can be manipulated to obtain permissible lengths and symmetry polarity for LPPRFB [28].

Theorem 1: For an M-channel LPPRFB with filter lengths $L_i=K_iM+\beta,\ 0\le\beta<M,\ K_i\ge1$, we have the following:

1) If M is even and β is even, there are $\frac{M}{2}$ symmetric and $\frac{M}{2}$ antisymmetric filters.
2) If M is even and β is odd, there are $(\frac{M}{2}+1)$ symmetric and $(\frac{M}{2}-1)$ antisymmetric filters.
3) If M is odd, there are $(\frac{M+1}{2})$ symmetric and $(\frac{M-1}{2})$ antisymmetric filters.

Notice that this theorem provides the most general constraint for LPPRFB's; it certainly holds for biorthogonal and PU systems. In other words, Theorem 1 requires an LPPRFB (with filters satisfying the stated length condition) to have the same number of symmetric and antisymmetric filters when the number of channel M is even and all the filters have even lengths. If M is even but all the filters are now odd length, then the system must have two more symmetric filters. For odd-channel systems, the number of symmetric filters always exceeds the number of antisymmetric filters by one. This is a useful and powerful result. It allows the designers of FB's to narrow down the search for possible solutions. It also helps to explain partially why only certain solutions exist. Before presenting the formal proof of the theorem, let us first go through a couple of useful lemmas that appear persistently throughout.

Lemma 1:

$$\mathrm{tr}(\hat{\mathbf{J}}(z)) = \begin{cases} 0 & \text{if } M \text{ is even and } \beta \text{ is even} \\ 1+z^{-1} & \text{if } M \text{ is even and } \beta \text{ is odd} \\ 1 & \text{if } M \text{ is odd and } \beta \text{ is even} \\ z^{-1} & \text{if } M \text{ is odd and } \beta \text{ is odd.} \end{cases} \tag{7}$$

Proof:

$$\mathrm{tr}(\hat{\mathbf{J}}(z)) = \mathrm{tr}\left(\begin{bmatrix} z^{-1}\mathbf{J}_\beta & \mathbf{0}_{\beta\times(M-\beta)} \\ \mathbf{0}_{(M-\beta)\times\beta} & \mathbf{J}_{M-\beta}\end{bmatrix}\right).$$

There are four cases to consider. When M is even and β is even, $(M-\beta)$ is also even, and all of the diagonal elements of the matrix $\hat{\mathbf{J}}(z)$ are all zeros. Thus, its trace is 0. When M is even and β is odd, $(M-\beta)$ is odd. Hence, we now pick up two nonzero elements on the diagonal of $\hat{\mathbf{J}}(z)$: 1 and z^{-1}. When M is odd, β and $(M-\beta)$ cannot be both odd or both even. Therefore, in this case, only one nonzero element can be picked up: either 1 or z^{-1}. □

Lemma 2:

$$|\hat{\mathbf{J}}(z)| = \left|\begin{bmatrix} z^{-1}\mathbf{J}_\beta & \mathbf{0}_{\beta\times(M-\beta)} \\ \mathbf{0}_{(M-\beta)\times\beta} & \mathbf{J}_{M-\beta}\end{bmatrix}\right| = \begin{cases} (-1)^{(\frac{M}{2}+\beta)}z^{-\beta} & \text{if } M \text{ is even} \\ (-1)^{(\frac{M-1}{2})}z^{-\beta} & \text{if } M \text{ is odd.}\end{cases} \tag{8}$$

Proof: Notice that $\hat{\mathbf{J}}(z)$ is a square block-diagonal matrix; thus, its determinant can be factorized as [9]

$$|\hat{\mathbf{J}}(z)| = |z^{-1}\mathbf{J}_\beta||\mathbf{J}_{M-\beta}| = z^{-\beta}|\mathbf{J}_\beta||\mathbf{J}_{M-\beta}|.$$

With the factorization above, coupled with the fact that $|\mathbf{J}_{4m}| = |\mathbf{J}_{4m+1}| = 1$ and $|\mathbf{J}_{4m+2}| = |\mathbf{J}_{4m+3}| = -1$, where m is any positive integer, one can verify that Lemma 2 holds by considering four possible cases: $M = 4m, M = 4m+1, M = 4m+2$, and $M = 4m+3$. □

With the help of Lemma 1, proving Theorem 1 is a trivial task.

Proof of Theorem 1: Since $\mathbf{E}(z)$ is invertible, and $\mathbf{D}^{-1} = \mathbf{D}$, (3) can be rewritten as

$$\mathbf{D} = \hat{\mathbf{Z}}(z)\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z)\mathbf{E}^{-1}(z). \tag{9}$$

Taking the trace of both sides and using the fact that $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$, one can obtain

$$\mathrm{tr}(\mathbf{D}) = \mathrm{tr}(\hat{\mathbf{Z}}(z)\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z)\mathbf{E}^{-1}(z)) = \mathrm{tr}(\mathbf{E}^{-1}(z)\hat{\mathbf{Z}}(z)\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z)).$$

$\mathrm{tr}(\mathbf{D})$ is a constant, and therefore, its value can be obtained by evaluating the right-hand side of the above equation at a specific value of the variable z. Since

$$\mathbf{E}(z^{-1})\hat{\mathbf{Z}}(z)\mathbf{E}^{-1}(z)|_{z=1} = \mathbf{E}(1)\mathbf{I}\mathbf{E}^{-1}(1) = \mathbf{I}$$

we have

$$\mathrm{tr}(\mathbf{D}) = \mathrm{tr}(\mathbf{E}^{-1}(z)\hat{\mathbf{Z}}(z)\mathbf{E}(z^{-1})\hat{\mathbf{J}}(z))|_{z=1} = \mathrm{tr}(\hat{\mathbf{J}}(z))|_{z=1}. \tag{10}$$

Again, there are four possible cases. Recall that $\mathbf{D}$ is a diagonal matrix whose entry is $+1$ when the corresponding filter is symmetric and -1 when the corresponding filter is antisymmetric. When both M and β are even, Lemma 1 yields $\mathrm{tr}(\mathbf{D}) = \mathrm{tr}(\hat{\mathbf{J}}(z))|_{z=1} = 0$. Hence, the system must have an equal number of symmetric and antisymmetric filters to satisfy the LP and PR properties. When M is even and β is odd, $\mathrm{tr}(\mathbf{D}) = (1+z^{-1})|_{z=1} = 2$. Thus, we need two more symmetric filters in this case. The results from the remaining two odd-M cases can be trivially obtained in a similar manner.

In time-domain FB designs [22], [23], the filters' symmetry polarity is not a narrow enough requirement. The filters' lengths are also very important for PR. If the designer chooses wrong filters' lengths, his optimization routine will not converge. Therefore, besides the necessary LP PR condition for the filters' symmetry polarity as stated in Theorem 1, we also have to obtain the necessary LP PR condition for the filters' lengths (more precisely speaking, the necessary condition for the sum of their lengths).

Theorem 2: For an M-channel LPPRFB with filter lengths $L_i = K_iM+\beta,\ 0 \le \beta < M,\ K_i \ge 1$, we have the following.

1) If M is even and β is even, $\sum_{i=0}^{M-1} K_i$ is even.
2) If M is even and β is odd, $\sum_{i=0}^{M-1} K_i$ is odd.
3) If M is odd and β is even, $\sum_{i=0}^{M-1} K_i$ is odd.
4) If M is odd and β is odd, $\sum_{i=0}^{M-1} K_i$ is even.

The proof of Theorem 2 based on Lemma 2 is presented in Appendix A. An interesting corollary can be derived on the behavior of the total lengths of all the filters in a LPPRFB.

Corollary 1: For an M-channel LPPRFB with filter lengths $L_i = K_iM+\beta,\ 0 \le \beta < M,\ K_i \ge 1$, and m is a positive interger, if M is even, $\sum_{i=0}^{M-1} L_i = 2mM$, and if M is odd, $\sum_{i=0}^{M-1} L_i = (2m+1)M$.

Proof: Since $L_i = K_iM+\beta$, $\sum_{i=0}^{M-1} L_i = M(\beta + \sum_{i=0}^{M-1} K_i)$. If M is even and β is also even, Theorem 2 requires $\sum K_i$ to be even. Hence, $(\beta + \sum_{i=0}^{M-1} K_i)$ is even. If β is now odd, then $\sum K_i$ has to be odd; therefore, $(\beta+\sum_{i=0}^{M-1} K_i)$ is even. For odd values of M, using a similar argument, we arrive at the conclusion that the total length of all the filters is an odd multiple of M. □

The results from Theorem 1, Theorem 2, and Corollary 1 are summarized in Table I. S stands for symmetric filters, and A stands for antisymmetric filters. Note that these results hold true for both sets of analysis and synthesis filters of any LP PR system satisfying (3). Similar results are also developed independently (but without proof) by Basu *et al.* in [24].

It is no surprise that the solutions for the well-studied two-channel LPPRFB agree with our result. There are two systems for two-channel LPFB's. The type A system has even-length filters with different symmetry polarity, whereas the Type B system has odd-length filters with the same symmetry. This can be confirmed using Table I. The type A system belongs to the first row ($M = 2$ and $\beta = 0$). Therefore, there must be one symmetric and one antisymmetric filter. Since they

TABLE I
POSSIBLE SOLUTIONS FOR M-CHANNEL LPPRFB WITH FILTERS OF ARBITRARY LENGTHS $L_i = K_i M + \beta$

Case	*Symmetry Polarity Condition*	*Lengths Condition*	*Sum of Lengths*
M even, β even	$\frac{M}{2}$ S & $\frac{M}{2}$ A	$\sum K_i$ even	$2mM$
M even, β odd	$(\frac{M}{2}+1)$ S & $(\frac{M}{2}-1)$ A	$\sum K_i$ odd	$2mM$
M odd, β even	$(\frac{M+1}{2})$ S & $(\frac{M-1}{2})$ A	$\sum K_i$ odd	$(2m+1)M$
M odd, β odd	$(\frac{M+1}{2})$ S & $(\frac{M-1}{2})$ A	$\sum K_i$ even	$(2m+1)M$

both have even length (and $K_0 + K_1$ is even), the sum of the filter lengths is a multiple of 4. Similarly, the type B system satisfies all constraints in the second row of the table (it belongs to the even-M, odd-β case with $M = 2$ and $\beta = 1$) [17]. Moreover, all of the M-channel solutions reported so far also follow the results in our two theorems. For example, the three-channel LPPRFB in [3] has two symmetric filters and one antisymmetric filter. They have lengths 56, 53, and 56 $(M = 3, \beta = 1)$. The sum of the corresponding K_i (18, 17, and 18) is odd. In addition, the total length is an odd multiple of 3. Another three-channel solution reported in [22] has two symmetric and one antisymmetric filter with lengths 53, 44, and 44, respectively. This system belongs to the case of odd M and even β. Therefore, the number of symmetric filters must exceed the number of antisymmetric ones by one. Moreover, the sum of K_i (17, 14, 14) is odd, which is consistent with the result in Theorem 2. Several LP cosine-modulated PR FB's with filter lengths not equal to KM have been reported recently in [14]. All of these FB lengths and polarity symmetry also fall within our constraints (the zero-value coefficients resulted from the optimization process need to be counted as well).

III. LATTICE STRUCTURE FOR EVEN-CHANNEL LPPUFB'S

In this section, a complete and minimal factorization for even-channel LPPUFBs will be presented. Lattice structures for even-channel systems with $\beta = 0$ have been reported in [1] and [2]. However, these structures impose a very strict restriction on both analysis and synthesis filters. They must have the same length, which is a multiple of the number of channels, i.e., $L = KM$. This restriction does not allow much flexibility in both of the system's design and implementation. Extending the filters' length from KM to $KM + \beta$ provides more degrees of freedom in fine-tuning the filters to meet certain specifications, i.e., stopband attenuation. In [1] and [2], the step size in increasing the filters' length is at least M. This is not so convenient when the number of channels M is large (say, 16 or 32). From a design point of view, a large increase in length means a much higher nonlinear parameter space to be searched, and the optimization program tends to be more easily trapped in local minima. From an implementation point of view, a large increase in filter length translates to a much higher computational complexity. This calls for LPPUFB's (GenLOT) with arbitrary-length filters. From the second entry of Table I, we know that an odd-length even-channel GenLOT does not exist. If all filters have the same odd length and M is even, then β is odd, and $K_i = K$. Hence, $\sum_{i=0}^{M-1} K_i = MK$, which has to be even, contradicting the requirement that $\sum_{i=0}^{M-1} K_i$ is odd. Thus, we can conclude that even-channel LPPUFB's with filters having the same length only exist if the filters' length is even. The length increment must be at least two taps at a time. In the lapped transform language, the number of overlapped samples must be even.

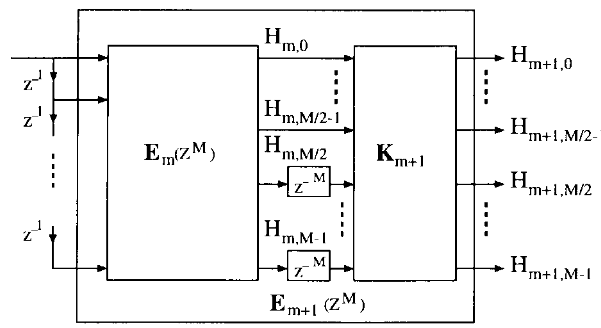


Fig. 4. Stage of the lattice structure pertaining to lattice structure factorization.

As we recall in Fig. 4, the main concept of lattice structure factorization is that given a set of filters with certain set of properties at the output, we would like to propagate this set of properties while reducing the filter length by M at each stage, i.e., peeling a block ($\mathbf{K}_{m+1}$) out. $\mathbf{K}_{m+1}$ must be chosen to satisfy the following properties.

1) Both sets of filters at stage m and stage $m+1$ have the same propagating properties.
2) The factorization has to be complete (i.e., for any choice of $H_{k+1}(z)$ satisfying these properties, there exists a peeling block) and minimal (in term of the number of delay elements used).
3) $\mathbf{K}_{m+1}$ is invertible. For PU systems, $\mathbf{K}_{m+1}$ is orthogonal.

From the first row of Table I, there are $\frac{M}{2}$ symmetric and $\frac{M}{2}$ antisymmetric filters in even-length even-channel LPPUFB's.

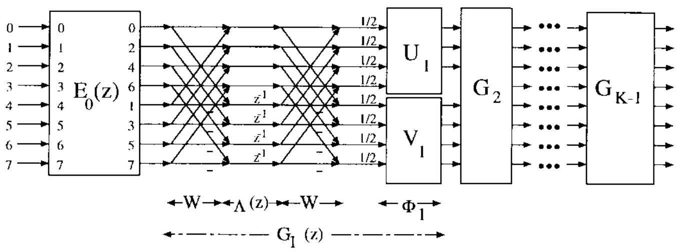


Fig. 5. General lattice structure for LPPUFB.

It can easily be shown that the same approach in [1] (propagating the pairwise time-reversed property) can be applied here to obtain the factorization for our PU polyphase matrix

$$\mathbf{E}(z) = \mathbf{S}\mathbf{K}_{K-1}\mathbf{\Lambda}(z)\mathbf{K}_{K-2}\mathbf{\Lambda}(z)\cdots\mathbf{K}_1\mathbf{\Lambda}(z)\mathbf{K}_0(z) \qquad (11)$$

where $\mathbf{K}_i,\ i = 1, 2, \ldots, K-1$ can be further factorized as

$$\begin{aligned}\mathbf{K}_i &= \frac{1}{2}\mathbf{Q}\mathbf{T}_i\mathbf{Q} = \frac{1}{2}\mathbf{Q}\mathbf{W}\mathbf{\Phi}_i\mathbf{W}\mathbf{Q} \\ &= \frac{1}{2}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_i\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}\end{bmatrix}\end{aligned} \qquad (12)$$

$\mathbf{S}$ can be written as

$$\mathbf{S} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{S}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_1\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{J}\\ \mathbf{I} & -\mathbf{J}\end{bmatrix} \qquad (13)$$

and $\mathbf{K}_0(z)$ is the polyphase matrix of any LP PU system with filters' length $M+\beta$. All submatrices $\mathbf{I}, \mathbf{J}, \mathbf{0}, \mathbf{S}_0, \mathbf{S}_1, \mathbf{U}_i, \mathbf{V}_i$, $i = 1, 2, \ldots, K-1$ are square matrices of size $\frac{M}{2}$; the latter four are any orthogonal matrices, and each can be completely parametrized by $\binom{M/2}{2}$ rotation angles.

The building blocks in (11) and (12) can be combined and rearranged to yield an equivalent factorization for teh LPPUFB called the GenLOT [2]. We can repeat the same process here to get the alternate factorization

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0(z) \qquad (14)$$

where

$$\begin{aligned}\mathbf{G}_i(z) &= \mathbf{\Phi}_i\mathbf{W}\mathbf{\Lambda}(z)\mathbf{W} \\ &= \frac{1}{2}\begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_i\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & z^{-1}\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix}.\end{aligned} \qquad (15)$$

Note that this alternate factorization, which is depicted in Fig. 5, propagates LP property instead of pairwise time-reversed property. It is also a better structure for implementation since it is modular. Now, the only concern is $\mathbf{E}_0(z)$, which is the starting block of the propagation. Notice that we have not presented a method to obtain $\mathbf{K}_0(z)$ in the earlier factorization. For the case $\beta = 0$, both $\mathbf{K}_0(z)$ and $\mathbf{E}_0(z)$ have no delay element, i.e., they are constant matrices [1], [2].

1) The Starting Block $\mathbf{E}_0(z)$*:* After each stage, the filters' length is increased by M because of the structure of $\mathbf{\Lambda}(z)$. In order to end up with the final length $L = KM + \beta$, we have to take care of the "extra" β coefficients in $\mathbf{E}_0(z)$, i.e., $\mathbf{E}_0(z)$ is the polyphase matrix of a LPPU system with filters' length $(M+\beta)$. (Of course, the "extra" β coefficients can be taken care of at the end as well. However, such structures will not be as modular.) Now, since $\mathbf{E}_0(z)$ must contain $\frac{M}{2}$ symmetric and $\frac{M}{2}$ antisymmetric filters, $\mathbf{E}_0$ has the form

$$\mathbf{E}_0(z) = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{S}_{00} + z^{-1}\mathbf{S}_{00}\mathbf{J} & \mathbf{S}_{01} & \mathbf{S}_{01}\mathbf{J}\\ \mathbf{A}_{00} - z^{-1}\mathbf{A}_{00}\mathbf{J} & \mathbf{A}_{01} & -\mathbf{A}_{01}\mathbf{J}\end{bmatrix} \qquad (16)$$

where submatrices $\mathbf{S}_{00}$ and $\mathbf{A}_{00}$ have size $\frac{M}{2} \times \beta$, whereas $\mathbf{S}_{01}$ and $\mathbf{A}_{01}$ have size $\frac{M}{2} \times \frac{M-\beta}{2}$. It can be verified that this form of $\mathbf{E}_0(z)$ allows the first β polyphases to have one order more than the remaining $M-\beta$ polyphases. In addition, since there must be $\frac{M}{2}$ symmetric and $\frac{M}{2}$ antisymmetric filters according to Table I, $\mathbf{S}_{00}$ and $\mathbf{A}_{00}$ have the same number of rows. The corresponding coefficient matrix $\mathbf{P}_0$ with each filter's impulse response arranged row-wise is

$$\mathbf{P}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{S}_{00} & \mathbf{S}_{01} & \mathbf{S}_{01}\mathbf{J} & \mathbf{S}_{00}\mathbf{J}\\ \mathbf{A}_{00} & \mathbf{A}_{01} & -\mathbf{A}_{01}\mathbf{J} & -\mathbf{A}_{00}\mathbf{J}\end{bmatrix}. \qquad (17)$$

In order for $\mathbf{E}_0(z)$ to be PU, $\mathbf{P}_0$ has to satisfy the following time-domain constraint [6], with $h_i(n)$ being its rows:

$$\sum_{n=-\infty}^{\infty} h_j(n)h_k^*(n-\ell M) = \delta(\ell)\delta(j-k). \qquad (18)$$

In matrix notation, it is equivalent to $\mathbf{E}_0(z)\tilde{\mathbf{E}}_0(z) = \mathbf{I}$, i.e.,

$$\begin{cases}\mathbf{S}_{00}\mathbf{S}_{00}^T + \mathbf{S}_{01}\mathbf{S}_{01}^T = \mathbf{I}\\ \mathbf{A}_{00}\mathbf{A}_{00}^T + \mathbf{A}_{01}\mathbf{A}_{01}^T = \mathbf{I}\end{cases} \begin{cases}\mathbf{S}_{00}\mathbf{J}\mathbf{S}_{00}^T = \mathbf{0}\\ \mathbf{A}_{00}\mathbf{J}\mathbf{A}_{00}^T = \mathbf{0}\\ \mathbf{S}_{00}\mathbf{J}\mathbf{A}_{00}^T = \mathbf{0}.\end{cases} \qquad (19)$$

The first two equations are referred to as the orthogonality conditions; the remaining three are referred to as the shift-orthogonality conditions. A simple solution for (19) was proposed in [28]—choosing any arbitrary $\frac{M}{2} \times \frac{M}{2}$ orthogonal matrix $\mathbf{U}_0$ and then inserting $\frac{\beta}{2}$ zero column(s) intermittently between the columns of $\mathbf{U}_0$ to form the $\frac{M}{2} \times \frac{M+\beta}{2}$ matrix $[\mathbf{S}_{00}\ \mathbf{S}_{01}]$ in (17). (The same procedure is used to obtain $[\mathbf{A}_{00}\ \mathbf{A}_{01}]$). More clearly, starting with an $\frac{M}{2} \times \frac{M}{2}$ orthogonal matrix $\mathbf{U}_0$ with columns $\mathbf{u}_i : \mathbf{U}_0 = [\mathbf{u}_1\ \ \mathbf{u}_2\ \ \cdots\ \ \mathbf{u}_{\frac{M}{2}}]$, we can insert $\frac{\beta}{2}$ zero columns alternately to obtain $[\mathbf{S}_{00}\ \mathbf{S}_{01}] = [\mathbf{u}_1\ \ \mathbf{0}\ \ \mathbf{u}_2\ \ \mathbf{0}\ \ \cdots\ \ \mathbf{u}_{\frac{M}{2}}]$.

It can easily be verified that this zero-column-inserting method yields four matrices $\mathbf{S}_{00}$, $\mathbf{S}_{01}$, $\mathbf{A}_{00}$, and $\mathbf{A}_{01}$ that satisfy (19). However, this solution is only general for the case when $\beta = 2$. Let $\mathbf{U}_{00}$ be an arbitrary matrix of size $\frac{M}{2} \times \frac{\beta}{2}$. A more general solution for shift-orthogonality in (19) is $\tilde{\mathbf{S}}_{00} \triangleq [\mathbf{U}_{00}\ \mathbf{\Gamma}_p\ \ \mathbf{U}_{00}\ \mathbf{\Gamma}_m]$, with $\mathbf{\Gamma}_p \triangleq (\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2)$, $\mathbf{\Gamma}_m \triangleq (\mathbf{\Gamma}_1 - \mathbf{\Gamma}_2)\mathbf{J}$, where $\mathbf{\Gamma}_1$, $\mathbf{\Gamma}_2$ are any two orthogonal matrices of size $\frac{\beta}{2}$. With the above form of $\tilde{\mathbf{S}}_{00}$, one can verify that $\tilde{\mathbf{S}}_{00}\mathbf{J}\tilde{\mathbf{S}}_{00}^T = \mathbf{0}$

$$\begin{aligned}&\tilde{\mathbf{S}}_{00}\mathbf{J}\tilde{\mathbf{S}}_{00}^T \\ &\quad= [\mathbf{U}_{00}\mathbf{\Gamma}_p \quad \mathbf{U}_{00}\mathbf{\Gamma}_m]\mathbf{J}\begin{bmatrix}\mathbf{\Gamma}_p^T\mathbf{U}_{00}^T\\ \mathbf{\Gamma}_m^T\mathbf{U}_{00}^T\end{bmatrix} \\ &\quad= [\mathbf{U}_{00}(\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2) \quad \mathbf{U}_{00}(\mathbf{\Gamma}_1 - \mathbf{\Gamma}_2)]\begin{bmatrix}(\mathbf{\Gamma}_1^T - \mathbf{\Gamma}_2^T)\mathbf{U}_{00}^T\\ \mathbf{J}(\mathbf{\Gamma}_1^T + \mathbf{\Gamma}_2^T)\mathbf{U}_{00}^T\end{bmatrix} \\ &\quad= \mathbf{U}_{00}\big[(\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2)(\mathbf{\Gamma}_1^T - \mathbf{\Gamma}_2^T) \\ &\qquad + (\mathbf{\Gamma}_1 - \mathbf{\Gamma}_2)(\mathbf{\Gamma}_1^T + \mathbf{\Gamma}_2^T)\big]\mathbf{U}_{00}^T = \mathbf{0}.\end{aligned} \qquad (20)$$

The remaining shift-orthogonality conditions can be verified similarly. For any arbitrary $\frac{M}{2} \times \frac{\beta}{2}$ matrix $\mathbf{U}_{00}$ and any arbitrary $\frac{M}{2} \times \frac{M-\beta}{2}$ matrix $\mathbf{U}_{01}$ (these matrix sizes guarantee the first β polyphase components to have an extra order), using the above solutions of shift-orthogonality, we can simplify and factorize $\mathbf{E}_0(z)$ as in (21), shown at the bottom of the page, with $\mathbf{I} \triangleq \mathbf{I}_{\frac{M-\beta}{2}}$.

The factorization of $\mathbf{E}_0(z)$ is shown in Fig. 6. This structure is minimal because it uses the least number of delays, in this case, $\beta/2$ (see Lemma 4 and its proof). However, it only guarantees shift-orthogonality so far. In order for $\mathbf{E}_0(z)$ to be PU, orthogonality needs to be imposed on $\mathbf{\Phi}_0$ and $\mathbf{\Gamma}$ as well. ($\mathbf{\Lambda}_0(z)$ is already PU). First of all, notice that $\mathbf{\Phi}_0$ is exactly the first block of GenLOT [2]; it can be further factorized as

$$\mathbf{\Phi}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{V}_{00} & \mathbf{V}_{01}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}}\\ \mathbf{I}_{\frac{M}{2}} & -\mathbf{I}_{\frac{M}{2}}\end{bmatrix} \times \begin{bmatrix}\mathbf{I}_{\frac{M}{2}} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\frac{M}{2}}\end{bmatrix}. \tag{22}$$

Thus, $\mathbf{\Phi}_0$ is orthogonal if $\mathbf{U}_0 \triangleq [\mathbf{U}_{00}\ \mathbf{U}_{01}]$ and $\mathbf{V}_0 \triangleq [\mathbf{V}_{00}\ \mathbf{V}_{01}]$ are orthogonal. Next, a permutation can turn $\mathbf{\Gamma}$ into

$$\tilde{\mathbf{\Gamma}} = \begin{bmatrix}\mathbf{\Gamma}_p & \mathbf{\Gamma}_m & \mathbf{0}\\ \mathbf{J}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{J}\mathbf{\Gamma}_p\mathbf{J} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{M-\beta}\end{bmatrix}.$$

Then, $\tilde{\mathbf{\Gamma}}$ can be further factorized as

$$\tilde{\mathbf{\Gamma}} = \begin{bmatrix}\mathbf{I}_{\frac{\beta}{2}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\frac{\beta}{2}} & \mathbf{I}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{I}_{\frac{\beta}{2}} & -\mathbf{I}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix} \times \begin{bmatrix}\frac{1}{2}\mathbf{\Gamma}_0 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \frac{1}{2}\mathbf{\Gamma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\frac{\beta}{2}} & \mathbf{I}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{I}_{\frac{\beta}{2}} & -\mathbf{I}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\frac{\beta}{2}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\frac{\beta}{2}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{23}$$

where all identity matrices $\mathbf{I}$ without the subscript have size $(M-\beta) \times (M-\beta)$. Again, $\tilde{\mathbf{\Gamma}}$ is orthogonal for arbitrary orthogonal matrices $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$. Combining (21)–(23), we have a factorization for the starting PU block $\mathbf{E}_0(z)$. The corresponding coefficient matrix $\hat{\mathbf{P}}_0$ can be shown to be (24), shown at the bottom of the page. By inspection, $\mathbf{E}_0(z)$ produces a LP system. Therefore, $\mathbf{E}_0(z)$ is LPPU, and $\mathbf{E}(z)$ in (14) is also LPPU. Now, to guarantee that no solution can be missed using this design procedure, the converse of this result, stated in the following theorem, has to be proven.

Theorem 3: The polyphase matrix $\mathbf{E}(z)$ of a LPPUFB with even length and even channel can always be factorized as in (14), where its factors are given by (15), (21)–(23) (see Fig. 7, which is drawn for $M=8$ and $\beta=4$). The lattice coefficients are the rotation angles of the orthogonal matrices $\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, \mathbf{U}_i$, and $\mathbf{V}_i$, $i=0,1,\ldots,K-1$. In addition, this factorization uses the minimum number of delays.

Proof: See Lemmas 3 and 4.

The total number of free parameters (lattice coefficients) to optimize is $2K\binom{M/2}{2} + 2\binom{\beta/2}{2}$. When β decreases to 0 or increases to M, the number of parameters changes consistently with those reported in [1] and [2] (reducing a stage or adding in a stage, respectively). It is also a simple exercise to show that when $M=8$, this lattice structure for $\mathbf{E}_0(z)$ reduces to the traditional DCT used in JPEG when $\beta=0$ and extends to the most complete LOT when $\beta=8$. □

Lemma 3: The proposed factorization of $\mathbf{E}(z)$ as in (14), where $\mathbf{E}_0(z)$ is given as in (21), is complete, i.e., it indeed spans the space of all even-channel even-length LP PU systems.

The proof of Lemma 3 is presented in Appendix B.

Lemma 4: The proposed factorization of $\mathbf{E}(z)$ as in (14), where $\mathbf{E}_0(z)$ is given as in (21), is minimal, i.e., it uses the minimal number of delays for its implementation.

Proof: A structure is said to be minimal if the number of delays used is equal to the degree of the transfer function. For a PU system, it has been proven in [6] that

$$\deg(\mathbf{E}(z)) = \deg(|\mathbf{E}(z)|).$$

$$\begin{aligned}\mathbf{E}_0(z) &= \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_{00}\mathbf{\Gamma}_p + z^{-1}\mathbf{U}_{00}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{U}_{00}\mathbf{\Gamma}_m + z^{-1}\mathbf{U}_{00}\mathbf{\Gamma}_p\mathbf{J} & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J}\\ \mathbf{V}_{00}\mathbf{\Gamma}_p - z^{-1}\mathbf{V}_{00}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{V}_{00}\mathbf{\Gamma}_m - z^{-1}\mathbf{V}_{00}\mathbf{\Gamma}_p\mathbf{J} & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J}\end{bmatrix}\\ &= \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J} & z^{-1}\mathbf{U}_{00}\mathbf{J}\\ \mathbf{V}_{00} & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J} & -z^{-1}\mathbf{V}_{00}\mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{\Gamma}_p & \mathbf{\Gamma}_m & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{J}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{J}\mathbf{\Gamma}_p\mathbf{J} & \mathbf{0} & \mathbf{0}\end{bmatrix}\\ &= \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_{00} & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J} & \mathbf{U}_{00}\mathbf{J}\\ \mathbf{V}_{00} & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J} & -\mathbf{V}_{00}\mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\frac{\beta}{2}} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & z^{-1}\mathbf{I}_{\frac{\beta}{2}}\end{bmatrix}\begin{bmatrix}\mathbf{\Gamma}_p & \mathbf{\Gamma}_m & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{J}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{J}\mathbf{\Gamma}_p\mathbf{J} & \mathbf{0} & \mathbf{0}\end{bmatrix}\\ &= \frac{1}{\sqrt{2}}\mathbf{\Phi}_0\mathbf{\Lambda}_0(z)\mathbf{\Gamma}.\end{aligned} \tag{21}$$

$$\hat{\mathbf{P}}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_{00}\mathbf{\Gamma}_p & \mathbf{U}_{00}\mathbf{\Gamma}_m & \mathbf{U}_{01} & \mathbf{U}_{01}\mathbf{J} & \mathbf{U}_{00}\mathbf{\Gamma}_m\mathbf{J} & \mathbf{U}_{00}\mathbf{\Gamma}_p\mathbf{J}\\ \mathbf{V}_{00}\mathbf{\Gamma}_p & \mathbf{V}_{00}\mathbf{\Gamma}_m & \mathbf{V}_{01} & -\mathbf{V}_{01}\mathbf{J} & -\mathbf{V}_{00}\mathbf{\Gamma}_m\mathbf{J} & -\mathbf{V}_{00}\mathbf{\Gamma}_p\mathbf{J}\end{bmatrix}. \tag{24}$$

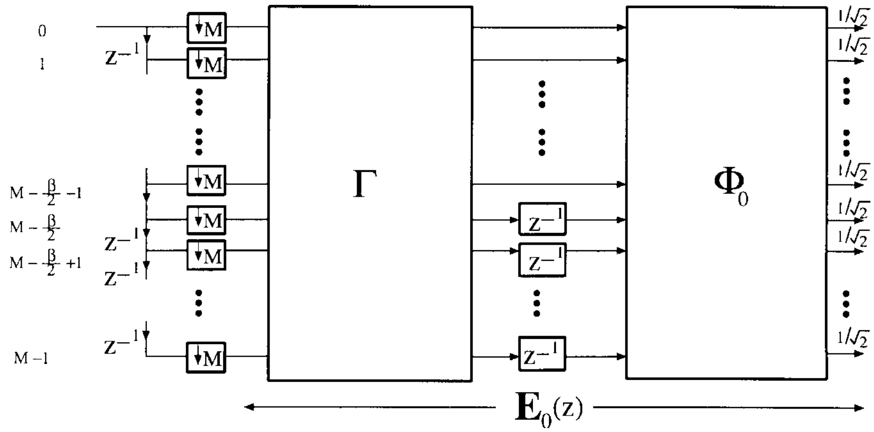


Fig. 6. Starting block $\mathbf{E}_0(z)$.

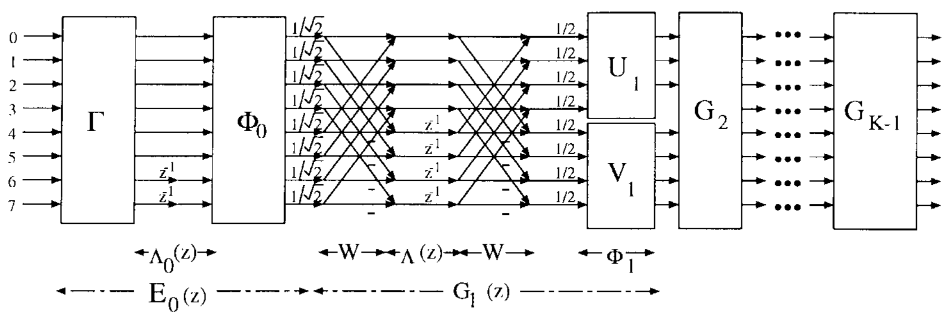


Fig. 7. Complete and minimal lattice structure for even-channel LPPUFB.

Using the symmetry property of the polyphase matrix in (3), we have

$$\begin{aligned}\deg(\mathbf{E}(z)) &= \deg(|\mathbf{E}(z)|)\\ &= \deg(|\mathbf{D}||z^{-(K-1)}||\mathbf{E}(z^{-1})||\hat{\mathbf{J}}(z)|).\end{aligned}$$

Therefore

$$\deg(\mathbf{E}(z)) = M(K-1) - \deg(\mathbf{E}(z)) + \beta$$

which leads to $\deg(\mathbf{E}(z)) = \frac{M(K-1)+\beta}{2}$. In our factorization, we use $\frac{M}{2}$ delays for each propagation block $\mathbf{G}_i(z)$ and $\frac{\beta}{2}$ delays for the starting block $\mathbf{E}_0(z)$, totalling the same number of $\frac{M(K-1)+\beta}{2}$ delays. Hence, the factorization is minimal. It is also interesting to note that when β increases to M, the number of delays in our structure increases to $\frac{MK}{2}$—a consistent number compared with the results in [1] and [2]. □

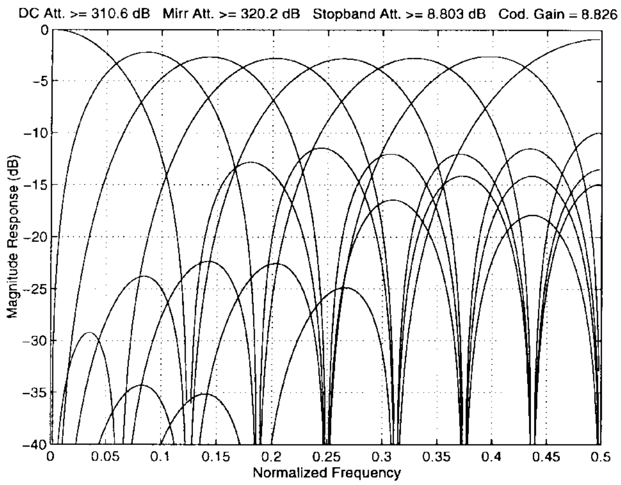
IV. Design Examples

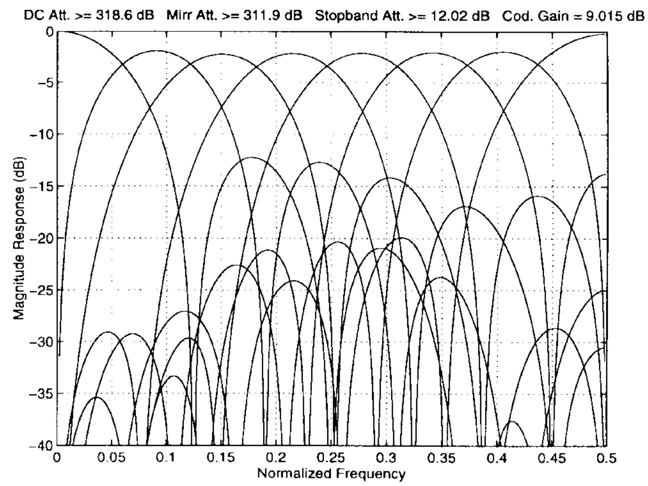
The lattice structure described in Section III is very simple to design because it fits perfectly in the GenLOT framework in [2]. The only difference is in the implementation of the starting block $\mathbf{E}_0(z)$. Fig. 8(b) shows a design example of an eight-channel LPPUFB with all filters having length 12 $(K = 1, \beta = 4)$. Refer to Fig. 6 for the implementation of this system. This new system can be thought of as a GenLOT with noninteger overlap; in this case, we have an overlap factor of $\frac{1}{2}$ (half-block overlap). For comparison purposes, the top left and bottom left of Fig. 8 shows the frequency responses of the well-known DCT $(K = 1, \beta = 0)$ and LOT $(K = 2, \beta = 0)$, respectively. Several other design examples are presented in Fig. 9. The $\beta = 0$ case (top right of Fig. 9) is included to serve as a comparison benchmark. All FB's presented in this paper are obtained from Matlab's nonlinear optimization routines with a general starting block. If DCT is desired here, the number of parameters is reduced by $2\binom{M/2}{2}$, leading to a narrower search and possibly sub-optimal systems.

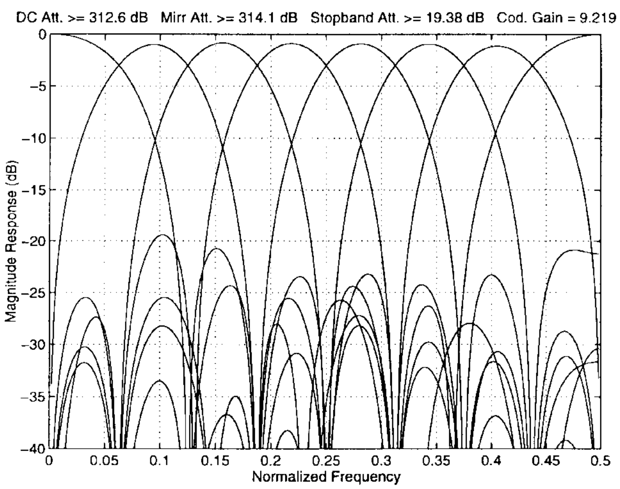
Finally, Fig. 10 illustrates the use of the FB's in Fig. 8 in image coding. To be fair, the same transform-based coder with optimal bit allocator, uniform scalar quantizer, run-length, and Huffman coder is used for all three cases. The differences lie at the transform and the blocking mechanism [26]. The botto right of Fig. 8 and Fig. 10 confirm that our design provides a new family of LPPUFB, which is very comparable with GenLOT. Interestingly enough, with only two more parameters to optimize and two more delays in the implementation, we are able to obtain a much improved LPPUFB compared with DCT. Objectively, all errors are lower. Subjectively, blocking artifacts are also reduced. The filters' coefficients of this new LOT length 12 and other design examples can be found at URL address http://saigon.ece.wisc.edu/~waveweb/QMF/GenlotDS.html.

V. Conclusion

In this paper, several important results are presented. Section II generalizes the symmetry property for the polyphase matrix







0.25 bpp	DCT	LOT L=12	LOT
MSE	268.6	249.3	223.7
PSNR	23.84 dB	24.16 dB	24.63 dB
MAX ERR.	111.6	88.44	82.60
# lattice coef.	12	14	24
# delays	0	2	4

Fig. 8. Frequency response comparison for LPPUFB I. (top left) Eight-channel length 8: DCT. (top right) Eight-channel length 12. (bottom left) Eight-channel length 16: LOT. (bottom right) Objective comparison for image coding example.

of arbitrary length LP systems. Exploiting this property, we derive several necessary constraints on the number of symmetric/antisymmetric filters and the filters' lengths of LP PR systems. These permissible symmetry polarity and lengths help the designers of FBs' narrow down their search for possible solutions. They also help us in deriving the lattice structure for all possible even-channel LPPUFB's. The lattice structure is proven to be complete, i.e., all even-channel LP PU systems can be realized by some combination of these lattice coefficients. We also prove that the proposed lattice structure is minimal in terms of the number of delays used for implementation. This is the true GenLOT, where the amount of overlap is not constrained to be a multiple of the number of channels. The permissible length constraint yields an elegant proof that the overlap has to be an even number of samples, i.e., odd-length GenLOT does not exist. The design is compatible with those in previous work [1], [2]; the difference lies in the starting block of the cascade structure. The new modular lattice guarantees LP and PU properties structurally, i.e., our system is still LP PU in spite of the quantization of the lattice coefficients (the rotation angles of the orthogonal matrices). The included design examples show that this structure can provide as good a LPPUFB as those reported previously in literature.

APPENDIX A
PROOF OF THEOREM 2

Case 1—M Is Even and β Is Even: The determinant of $\mathbf{D}$ can also be manipulated in the same manner as its trace to prove Theorem 2. Taking the determinant of both sides of (3) gives

$$\begin{aligned}|\mathbf{E}(z)| &= |\mathbf{D}||\hat{\mathbf{Z}}(z)||\mathbf{E}(z^{-1})||\hat{\mathbf{J}}(z)| \\ &= |\mathbf{D}|z^{-\left(\sum_{i=0}^{M-1} K_i\right)+M}|\mathbf{E}(z^{-1})||\hat{\mathbf{J}}(z)| \quad \text{(A.1)}\end{aligned}$$

where we have used the fact that the determinant of the product of two square matrices is equal to the product of the determinants of the factors [9]. Evaluating (A.1) at $z = 1$ gives $|\mathbf{D}||\hat{\mathbf{J}}(1)| = 1$. Using the result from Lemma 2 to substitute for $|\hat{\mathbf{J}}(1)|$, one can see that for even M and even β, the relation

$$|\mathbf{D}||\hat{\mathbf{J}}(1)| = |\mathbf{D}|(-1)^{(\frac{M}{2}+\beta)} = |\mathbf{D}|(-1)^{(\frac{M}{2})} = 1$$

must hold. This is consistent with the derivation from the trace previously. When $\frac{M}{2}$ is odd, $|\mathbf{D}|$ must be -1, i.e., there are an

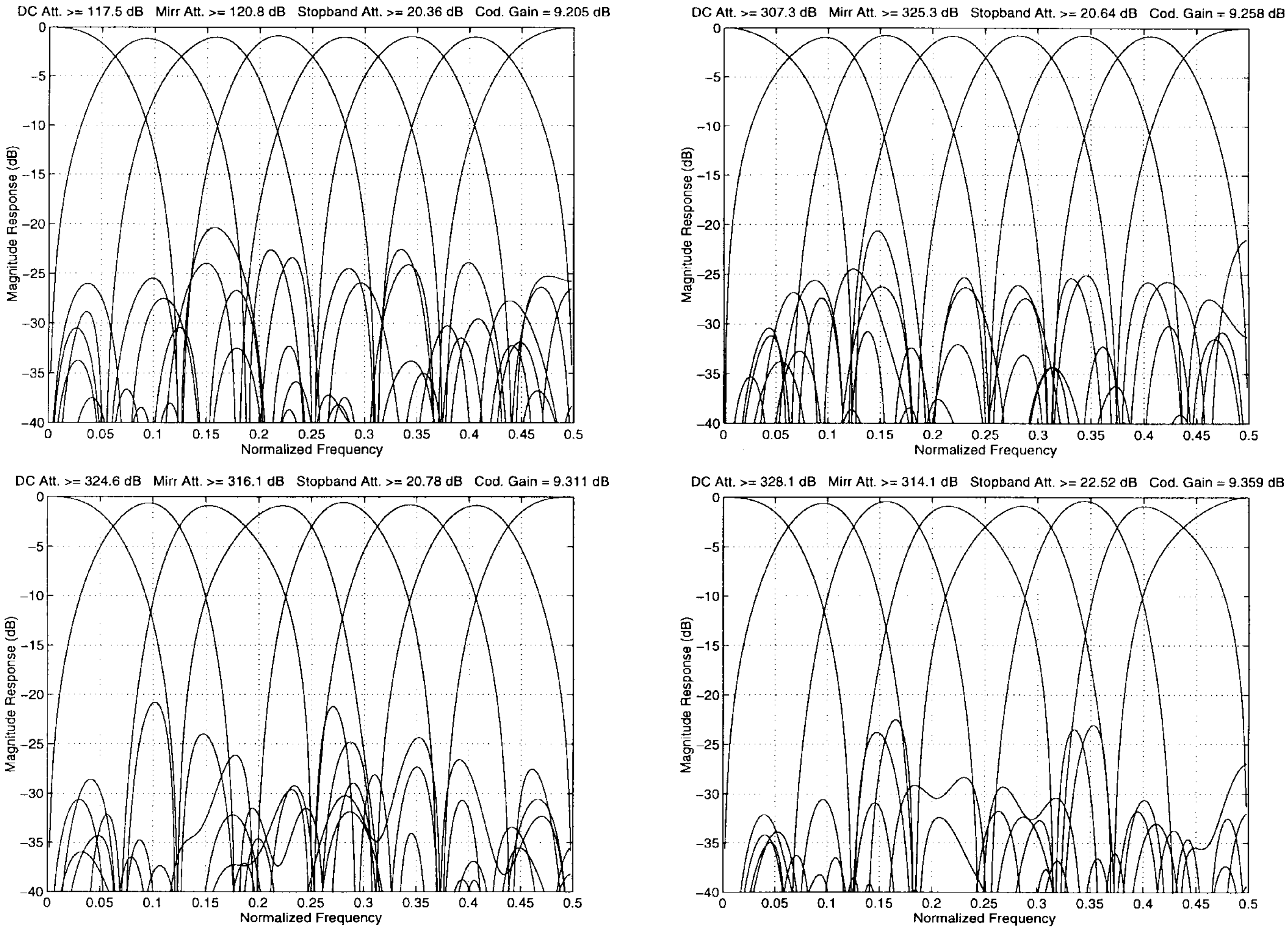


Fig. 9. Frequency response comparison for LPPUFB II. (top left) Eight-channel length 28. (top right) Eight-channel length 32. (bottom left) Eight-channel length 34. (bottom right) Eight-channel length 38.

odd number of antisymmetric filters. On the other hand, when $\frac{M}{2}$ is even, $|\mathbf{D}|$ must be $+1$ because now, there are an even number of antisymmetric filters.

Therefore, by evaluating the determinant of $\mathbf{D}$ at $z = 1$, nothing is gained, i.e., one can only confirm the validity of result from Theorem 1. However, evaluating (A.1) at $z = -1$ gives

$$\begin{aligned}(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}&|\mathbf{D}||\hat{\mathbf{J}}(-1)| \\ &= (-1)^{-(\sum_{i=0}^{M-1} K_i)}|\mathbf{D}|(-1)^{(\frac{M}{2}+\beta)}(-1)^{-\beta} = 1 \quad \text{(A.2)}\end{aligned}$$

where the result from Lemma 2 is used to substitute for $|\hat{\mathbf{J}}(-1)|$. Note that this is true for all cases with even M. Since β is also even in this case, (A.2) simplifies to

$$(-1)^{-(\sum_{i=0}^{M-1} K_i)}|\mathbf{D}|(-1)^{\frac{M}{2}} = 1.$$

For even M, there are two cases to consider: If $M = 4m$, then $\frac{M}{2}$ is even, and $|\mathbf{D}| = 1$; therefore, the sum $\sum_{i=0}^{M-1} K_i$ has to be even; similarly, if $M = 4m+2$, then $\frac{M}{2}$ is odd $|\mathbf{D}| = -1$, and $\sum_{i=0}^{M-1} K_i$ must be even.

Case 2—M Is Even and β Is Odd: At $z = 1$, following the same derivation as in *Case 1*, we get

$$|\mathbf{D}||\hat{\mathbf{J}}(1)| = |\mathbf{D}|(-1)^{(\frac{M}{2}+\beta)} = 1$$

where it can easily be verified that $|\mathbf{D}|$ is consistent with the result in Theorem 1. At $z = -1$, with β odd, (A.2) simplifies to

$$(-1)^{-(\sum_{i=0}^{M-1} K_i)}|\mathbf{D}|(-1)^{(\frac{M}{2}+\beta)} = -1.$$

Again, there are two cases to consider: $M = 4m$ and $M = 4m+2$. When $M = 4m$, $\frac{M}{2}$ is even, and the above equation reduces to

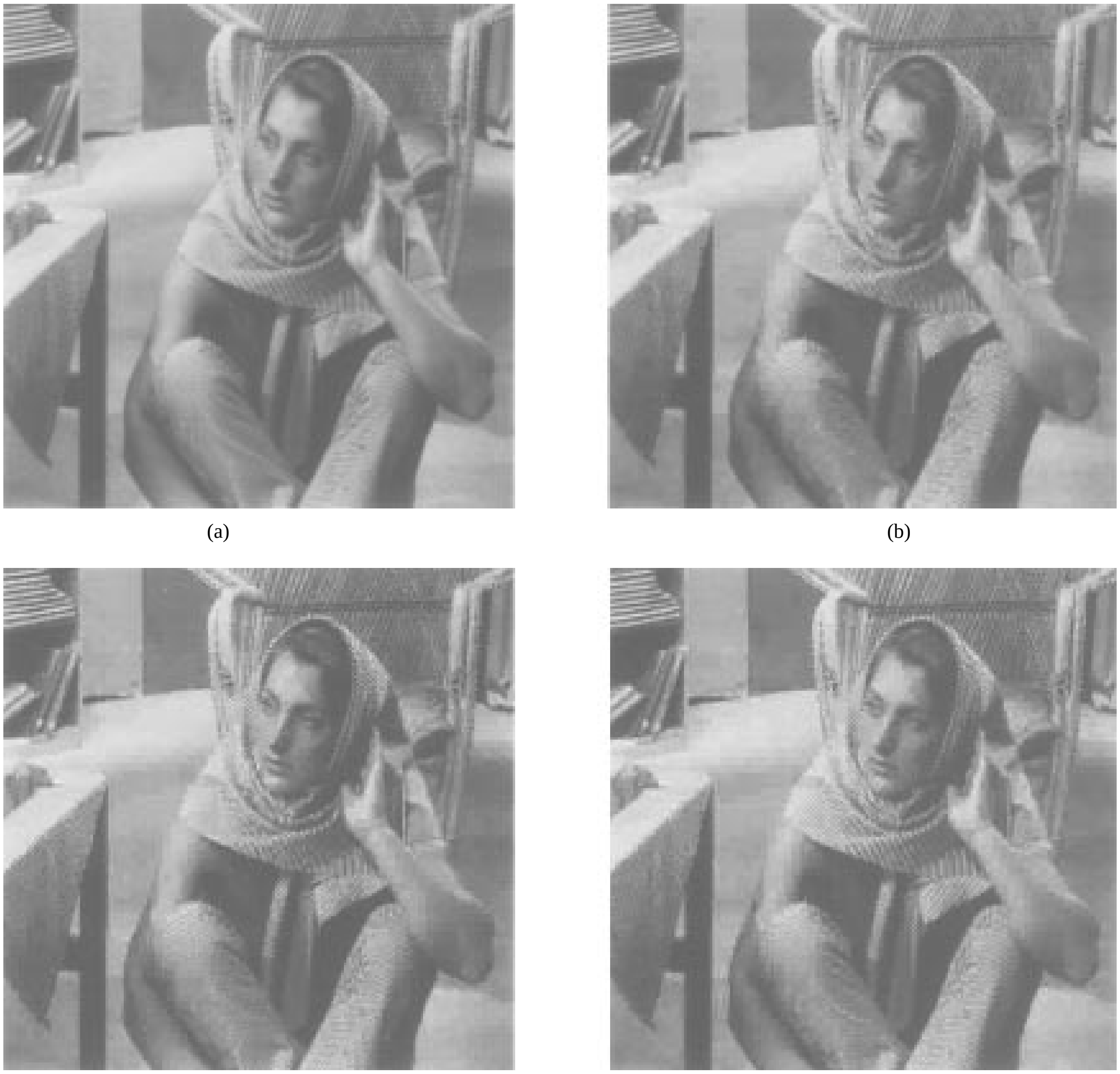
$$(-1)^{-(\sum_{i=0}^{M-1} K_i)}|\mathbf{D}| = 1.$$

In addition, for $M = 4m$, Theorem 1 requires an odd number of antisymmetric filters, implying $|\mathbf{D}| = -1$. Hence, $\sum_{i=0}^{M-1} K_i$ must be odd.

When $M = 4m+2$, $\frac{M}{2}$ is odd; therefore

$$(-1)^{-(\sum_{i=0}^{M-1} K_i)}|\mathbf{D}| = -1.$$

For $M = 4m + 2$, Theorem 1 says there must be an even number of antisymmetric filters (either $2m$ or $2m+2$), implying $|\mathbf{D}| = 1$. Then, $\sum_{i=0}^{M-1} K_i$ has to be odd.



(a) (b) (c) (d)

Fig. 10. Image coding example at 0.25 b/pixel: (a) Original image. (b) Coded image using DCT (eight-channel length 8). (c) Coded image using new LPPUFB (eight-channel length 12). (d) Coded image using LOT (eight-channel length 16).

Case 3—M Is Odd and β Is Even: Evaluating (A.1) at $z=-1$ and substituting the result from Lemma 2 for $|\hat{\mathbf{J}}(-1)|$, we get

$$\begin{aligned}(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}&|\mathbf{D}||\hat{\mathbf{J}}(-1)|\\ &=(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}|\mathbf{D}|(-1)^{(\frac{M-1}{2})}(-1)^{-\beta}=1.\end{aligned}\tag{A.3}$$

Since β is even, (A.3) simplifies to

$$(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}|\mathbf{D}|(-1)^{(\frac{M-1}{2})}=1. \tag{A.4}$$

For $M=4m+1$, $\frac{M-1}{2}$ is even. Therefore, $(-1)^{(\frac{M-1}{2})}=1$. Moreover, the number of antisymmetric filters is also even, implying $|\mathbf{D}|=1$. Therefore, $(\sum_{i=0}^{M-1} K_i-M)$ must be even for (A.4) to hold. It follows that $\sum_{i=0}^{M-1} K_i$ has to be odd. For $M=4m+3$, $\frac{M-1}{2}$ is odd. Therefore, $|\mathbf{D}|=-1$, $(-1)^{(\frac{M-1}{2})}=-1$, and

$$\begin{aligned}(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}&|\mathbf{D}|(-1)^{(\frac{M-1}{2})}\\ &=(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}=1.\end{aligned}$$

This implies that $(\sum_{i=0}^{M-1} K_i-M)$ must be even, and $\sum_{i=0}^{M-1} K_i$ has to be odd.

Case 4—M Is Odd and β Is Odd: With odd β, (A.3) can be simplified to

$$(-1)^{-(\sum_{i=0}^{M-1} K_i)+M}|\mathbf{D}|(-1)^{(\frac{M-1}{2})}=-1. \tag{A.5}$$

For $M=4m+1$, $\frac{M-1}{2}$ is even, and $(-1)^{(\frac{M-1}{2})}=1$. Moreover, the number of antisymmetric filters is also even, implying that $|\mathbf{D}|=1$. Thus, $(\sum_{i=0}^{M-1} K_i-M)$ must be odd for the above equation to hold. Therefore, $\sum_{i=0}^{M-1} K_i$ is even. For $M=4m+3$, $\frac{M-1}{2}$ is odd. Hence, $|\mathbf{D}|=-1$, $(-1)^{(\frac{M-1}{2})}=-1$, and the same result can be obtained: $\sum_{i=0}^{M-1} K_i$ is even.

APPENDIX B
PROOF OF LEMMA 3

This lemma can be rephrased as follows: Suppose there exists an arbitrary FIR LP PU matrix $\mathbf{E}(z)$, satisfying (3); then $\mathbf{E}(z)$ can always be factored as in (14), whereas $\mathbf{E}_0(z)$ can always be factored as in (21).

The former is achieved by performing the *order reduction* process in a similar procedure as presented in [1]. The factorization in (11) was proven complete, i.e., there exists lattice structure in the proposed from which we retain the pairwise time-reversed property, the PU property, and the causal property such that the order of $\mathbf{E}(z)$ is reduced by 1 after each stage. The alternate modular factorization in (14) is a rearrangement of the building blocks in (11); therefore, it is also complete [2], i.e., there exists lattice structure as in (14), which retains the LP property, the PU property, and the causal property such that the order of $\mathbf{E}(z)$ is reduced by 1 after each stage. Given a polyphase matrix with filters of length $L = KM + \beta$, after $K - 1$ reduction steps performed by $\mathbf{G}_i(z)$, $i = 1, 2, \ldots, K - 1$, the remainder is the LP PU system $\mathbf{E}_0(z)$ as shown in (16). Now, all what is left to prove is the latter part of the lemma: $\mathbf{E}_0(z)$ can always be factored as in (21).

Given a starting block $\mathbf{E}_0(z)$ as in (16), (17) shows that the corresponding coefficient matrix $\mathbf{P}_0$ will take the form in (17), where $\mathbf{S}_{00}$, $\mathbf{S}_{01}$, $\mathbf{A}_{00}$, and $\mathbf{A}_{01}$ must satisfy the shift-orthogonality and orthogonality condition in (19), respectively. On the other hand, from the proposed factorization of $\mathbf{E}_0(z)$, the corresponding coefficient matrix $\hat{\mathbf{P}}_0$ takes the form in (24).

Now, we have to prove that there exists orthogonal matrices $\mathbf{U}_0 \triangleq [\mathbf{U}_{00}\ \mathbf{U}_{01}]$, $\mathbf{V}_0 \triangleq [\mathbf{V}_{00}\ \mathbf{V}_{01}]$ of size $\frac{M}{2} \times \frac{M}{2}$, and $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$ of size $\frac{\beta}{2} \times \frac{\beta}{2}$ such that

$$\mathbf{U}_{01} = \mathbf{S}_{01}, \quad [\mathbf{U}_{00}(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1) \quad \mathbf{U}_{00}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_1)\mathbf{J}] = \mathbf{S}_{00}$$

and similarly

$$\mathbf{V}_{01} = \mathbf{A}_{01}, \quad [\mathbf{V}_{00}(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1) \quad \mathbf{V}_{00}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_1)\mathbf{J}] = \mathbf{A}_{00}.$$

The proof for existence of $\mathbf{U}_{01}$ and $\mathbf{V}_{01}$ is not difficult. By imposing the PU constraint on $\mathbf{E}_0(z)$ in (16) $\tilde{\mathbf{E}}_0(z)\mathbf{E}_0(z) = z^{-l}\mathbf{I}$, it can be shown that the columns of $\mathbf{S}_{01}$ must be orthonormal, i.e., $\mathbf{S}_{01}^T\mathbf{S}_{01} = \mathbf{I}_{\frac{M-\beta}{2}}$. Since $\mathbf{U}_{01}$ includes $\frac{M-\beta}{2}$ columns of an arbitrary orthogonal $\frac{M}{2} \times \frac{M}{2}$ matrix $\mathbf{U}_0$, $\mathbf{U}_{01}$ surely spans the space of all possible $\mathbf{S}_{01}$. A similar argument can be constructed for $\mathbf{V}_{01}$ and $\mathbf{A}_{01}$.

The proof for existence of the remaining building blocks is a little more tricky. We have to show that $[\mathbf{U}_{00}(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1)\ \mathbf{U}_{00}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_1)\mathbf{J}]$ spans the space of all possible $\mathbf{S}_{00}$. Since $\mathbf{S}_{00}\mathbf{S}_{00}^T + \mathbf{S}_{01}\mathbf{S}_{01}^T = \frac{1}{2}\mathbf{I}_{\frac{M}{2}}$, $[\mathbf{S}_{00} \quad \mathbf{S}_{01}]$ has rank $\frac{M}{2}$, i.e., the matrix has $\frac{M}{2}$ independent columns out of its $\frac{M+\beta}{2}$ columns. Moreover, shift-orthogonality must also be satisfied, i.e., $\mathbf{S}_{00}\mathbf{J}\mathbf{S}_{00}^T = \mathbf{0}_{\frac{M}{2}}$. This means that the columns of $\mathbf{J}\mathbf{S}_{00}^T$ lie in the nullspace of $\mathbf{S}_{00}$. However, $\text{rank}(\mathbf{J}\mathbf{S}_{00}^T) = \text{rank}(\mathbf{S}_{00}^T) = \text{rank}(\mathbf{S}_{00}) =$ dimension of the nullspace of $\mathbf{S}_{00}$. Since for any $\frac{M}{2} \times \beta$ matrix $\mathbf{S}_{00}$

$$\text{dimension of column space} + \text{dimension of nullspace} = \beta.$$

The dimension of column space of $\mathbf{S}_{00}$ must be $\frac{\beta}{2}$, or in other words, $\mathbf{S}_{00}$ must have $\frac{\beta}{2}$ independent columns. As a result, all $\frac{M-\beta}{2}$ columns of $\mathbf{S}_{01}$ must be independent. This agrees with our result. Recall that $\mathbf{U}_0 \triangleq [\mathbf{U}_{00}\ \mathbf{U}_{01}]$ is an $\frac{M}{2} \times \frac{M}{2}$ orthogonal matrix with $\mathbf{U}_{00}$ containing the first $\frac{\beta}{2}$ columns and $\mathbf{U}_{01}$ containing the last $\frac{M-\beta}{2}$ columns. Since the columns of any orthogonal matrix are independent,

$$\text{rank}(\mathbf{U}_{01}) = \frac{M-\beta}{2} = \text{rank}(\mathbf{S}_{01})$$

or $\mathbf{U}_{01}$ spans the space of all possible $\mathbf{S}_{01}$. Similarly, since $\text{rank}(\mathbf{U}_{00}) = \frac{\beta}{2}$

$$\text{rank}([\mathbf{U}_{00}(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1) \quad \mathbf{U}_{00}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_1)\mathbf{J}]) = \frac{\beta}{2} = \text{rank}(\mathbf{S}_{00})$$

leading to the same conclusion that $[\mathbf{U}_{00}(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1)\ \mathbf{U}_{00}(\mathbf{\Gamma}_0 - \mathbf{\Gamma}_1)\mathbf{J}]$ spans the space of all possible $\mathbf{S}_{00}$. The same proof can be conducted for $\mathbf{V}_0$, $\mathbf{A}_{00}$, and $\mathbf{A}_{01}$.

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