Department of Electrical and Computer Engineering
The Johns Hopkins University
520.137 Introduction to Electrical and Computer Engineering – Fall 2018

Homework Assignment IX

Reading Assignment: Kuc Chapter 7 and Lecture Notes

1. Consider the following $(5,2)$ ECC code which contains a mixture of parity and repetitive bits:

$$M_1 M_2 P_1 P_2 P_3$$

where the two repetitive bits are $P_1 = M_1$ and $P_2 = M_2$ whereas the parity bit is $P_3 = M_1 \oplus M_2$.

   (a) Construct a table of all valid code-words.

   (b) What is the minimum Hamming distance between the valid code-words? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?

   (c) Another $(5,2)$ ECC code is constructed as follows:

$$M_1 M_2 P_1 P_2 P_3$$

   where $P_1 = M_1 \oplus M_2$, $P_2 = M_1 M_2$, and $P_3 = M_1 + M_2$.
   Again, construct a table of all valid code-words.

   (d) What is the minimum Hamming distance for this ECC code? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?

   (e) A purely repetitive $(6,2)$ ECC code is constructed by simply repeating each message bit twice. Construct a table of all valid code-words for this $(6,2)$ repetitive code.

   (f) What is the minimum Hamming distance for this ECC code? What is the maximum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?

   (g) Among the 3 ECC codes explored in Part (a), (c) and (e), which one is the best? Justify briefly.

2. Consider the design of a Hamming-style $(5,3)$ parity-checking code. Each valid code-word has the following form:

$$X_3 X_2 X_1 P_2 P_1$$

where the 2 parity bits are $P_2 = X_2 \oplus X_3$ and $P_1 = X_1 \oplus X_2$.

   (a) Construct a table of all valid code-words.

   (b) What is the minimum Hamming distance between the valid code-words? What is the maximum Hamming distance for this code?

   (c) How many bit error(s) can we detect in this case? How many bit error(s) can we correct?

   (d) Can we improve the detection/correction capability by adding one additional parity bit as follows: $P_3 = X_3 \oplus X_1$? Why or why not?

3. Consider the following $(7,4)$ error-resilient code below:

$$M_1\ M_2\ M_3\ M_4\ P_1\ P_2\ P_3$$

where

$$P_1 = M_1 \oplus M_3 \text{ is the parity-check bit for the odd-indexed message-bit,}$$
$$P_2 = M_2 \oplus M_4 \text{ is the parity-check bit for the even-indexed message-bit,}$$
$$P_3 = P_1 \oplus P_2 \text{ offers a second layer of protection on the two parity bits.}$$

(a) Construct a table of all valid code-words. How many valid code-words are there? How many possible invalid code-words are there?

(b) What is the maximum Hamming distance between the valid code-words? What is the minimum Hamming distance between the valid code-words? How many bit-error(s) can we detect? How many bit-error(s) can we correct?

(c) Design the error-detection circuit for this redundant code using basic gates.

(d) If we replace the third parity-bit as follows

$$P_3 = M_1 \oplus M_2,$$

and we add a fourth parity bit

$$P_4 = M_3 \oplus M_4,$$

can we improve the error detection capability? Can we improve the error correction capability improved? Explain.

(e) Implement the modified encoder in Part (d) with basic gates.

Due date: **Dec. 5 in class**