Department of Electrical and Computer Engineering
The Johns Hopkins University
520.643 Digital Multimedia Coding and Processing – Spring 2003

Homework Assignment I

1. Let $\mathcal{X}$ be a random variable distributed over the set $\{x_1, x_2, \ldots, x_N\}$ with associated probabilities $\{P_1, P_2, \ldots, P_N\}$.

   a. Show that the entropy H of $\mathcal{X}$ satisfies $0 \le H \le \log N$.

   b. Find the necessary and sufficient conditions under which equality holds.

2. Let X denote a random variable distributed on the set $\{x_1, x_2, \ldots, x_N\}$ with associated probabilities $\{P_1, P_2, \ldots, P_N\}$. Let Y be another random variable defined on the same set but distributed uniformly. Show that
$$H(X) \le H(Y)$$
with equality if and only if X is also uniformly distributed.
Hint : First prove the inequality $\ln w \le w - 1$ with equality for $w = 1$, then apply this inequality to $\sum_{n=1}^{N} P_n \ln \frac{1/N}{P_n}$.

3. Design a Huffman code for a source with n output symbols and corresponding probabilities $\{\frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{2^{n-1}}, \frac{1}{2^{n-1}}\}$. Show that the average codeword length for such a source is equal to the source entropy.

4. Show that $\{01, 100, 101, 1110, 1111, 0011, 0001\}$ cannot be a Huffman code for any source probability distribution. Suggest appropriate modifications in the code so that it becomes a legitimate Huffman code.

5. **Computer Assignment**. Go to the course webpage

http://thanglong.ece.jhu.edu/Course/643/

and download the QCIF sequence named `glasgow100.qcif`. Please also read the `format.txt` file to understand how a typical raw YCrCb video sequence is stored. Each pixel is represented by 1 byte (`unsigned char` in C) and takes on values $0 - 255$. You can play these sequences using `splay.c`, `YUVplayer.exe`, or `CIFplayer.exe` – all available on the course webpage. A nice and simple image viewer `IrfanView` is also available. The file `utility.c` contains some simple routines that you may find useful.

a. Write a computer program to extract any luminance frame and output it as a `pgm` image given the video data and the frame index as inputs. The `pgm` format is a raw gray-scale image file with the following header:
P5
width height
255
where width and height are 176 and 144, respectively, in QCIF format.

b. Write a program to compute the entropies, the mean-square error, the peak signal-to-noise ratio, the mean absolute difference, and the maximum pixel error between any two given images or video frames.

c. Use the program above to compute the PSNRs between frame 0 and frames 1 to 30 of the *Glasgow* sequence. Plot the PSNRs as a function of time (index) difference. What do you observe from your plot?

Due date: **Feb. 21** in class