Department of Electrical and Computer Engineering
The Johns Hopkins University
520.643 Digital Multimedia Coding and Processing – Spring 2003

Homework Assignment III

1. (10 points) *Parseval's relation for orthonormal basis.* Consider the space $\mathcal{V} = \mathcal{R}^n$ and an orthonormal basis $\{\phi_i\}$. Prove that for every $\mathbf{v} \in V$,

$$\parallel \mathbf{v} \parallel^2 = \sum_{i=0}^{n-1} | < \phi_i, \mathbf{v} > |^2.$$

2. (30 points) *Parseval's relation for nonorthogonal basis.* Consider the same space $\mathcal{V} = \mathcal{R}^n$ with a biorthogonal basis, that is, two sets $\{\alpha_i\}$ and $\{\beta_i\}$ such that

$$< \alpha_i, \beta_j > = \delta[i - j], \quad i, j = 0, 1, \ldots, n - 1.$$

a. Show that any vector $\mathbf{v} \in V$ can be written in the following two ways:

$$\mathbf{v} \;=\; \sum_{i=0}^{n-1} < \alpha_i, \mathbf{v} > \; \beta_i \;=\; \sum_{i=0}^{n-1} < \beta_i, \mathbf{v} > \; \alpha_i.$$

b. Call $\mathbf{v}_\alpha$ the vector with entries $< \alpha_i, \mathbf{v} >$ and $\mathbf{v}_\beta$ the vector with entries $< \beta_i, \mathbf{v} >$. Given $\parallel \mathbf{v} \parallel$, what can you say about $\parallel \mathbf{v}_\alpha \parallel$ and $\parallel \mathbf{v}_\beta \parallel$?

c. Show that the generalization of *Parseval's identity* to biorthogonal systems is

$$\parallel \mathbf{v} \parallel^2 = \; | < \mathbf{v}, \mathbf{v} > | \; = \; | < \mathbf{v}_\alpha, \mathbf{v}_\beta > |.$$

and

$$< \mathbf{u}, \mathbf{v} > = < \mathbf{u}_\alpha, \mathbf{v}_\beta > .$$

3. (20 points) Find the autocorrelation matrix of a zero-mean WSS $AR(1)$ process with autocorrelation coefficient ρ.

4. **Computer Assignment** (40 points). Implement a closed-loop DPCM video coder with uniform scalar quantizer. Use the Huffman coder to encode the quantized predicted sample. You should also provide a lossless mode where there is no quantization at all.

   Test your simple video coder on the *glasgow* sequence. How much compression can you achieve losslessly? Report the PSNR's at several lossy compression ratios.

Due date: **Mar 21** in class