Department of Electrical and Computer Engineering
The Johns Hopkins University
520.643 Digital Multimedia Coding and Processing – Spring 2003

Homework Assignment IV

1. (30 points) The normalized Type-II forward and inverse M-point discrete cosine transform (DCT) is defined below

$$
\begin{aligned}
X[m] &= \sqrt{\frac{2}{M}}\, c_m \sum_{n=0}^{M-1} x[n] \; cos\left[\frac{(2n+1)m\pi}{2M}\right], \quad m = 0, 1, \ldots, M-1 \\
x[n] &= \sqrt{\frac{2}{M}} \sum_{m=0}^{M-1} c_m X[m] \; cos\left[\frac{(2n+1)m\pi}{2M}\right], \quad n = 0, 1, \ldots, M-1
\end{aligned}
$$

where

$$
c_m = \begin{cases} \frac{1}{\sqrt{2}} & m = 0 \\ 1 & m \neq 0. \end{cases}
$$

Write Matlab functions to plot the 1D and 2D basis functions of the $M \times M$ DCT. Plot the 1D and 2D basis functions of the 4-point and 8-point DCT.

Also write a Matlab function to plot the frequency responses of the DCT basis functions. What do you observe from the frequency responses? How do the basis functions partition the frequency spectrum?

2. (40 points) Implement in Matlab the forward and inverse 8×8 2D DCT directly: blocking the input image into 8×8 non-overlapping blocks $\mathbf{b}_{ij}$ and transform each of them independently using matrix multiplication, i.e., $\mathbf{B}_{ij} = \mathbf{C}\mathbf{b}_{ij}\mathbf{C}^T$ where $\mathbf{C}$ is the DCT transform matrix. Compute the PSNR between the original and the reconstructed image without any quantization? Comment on the transform's perfect reconstruction.

   Implement in C the fast 8×8 forward DCT as shown in Figure 1. Implement the fast inverse DCT by reversing the signal flow. Note that the transform coefficients $X[i]$ in Figure 1 are scaled up by a factor of 2. Compute the PSNR between the original and the reconstructed image. You can take a look at the JPEG codes at *www.ijg.org* and *www.jpeg.org.* Compare the two implementations and make sure that they yield the same DCT coefficients for any input image.

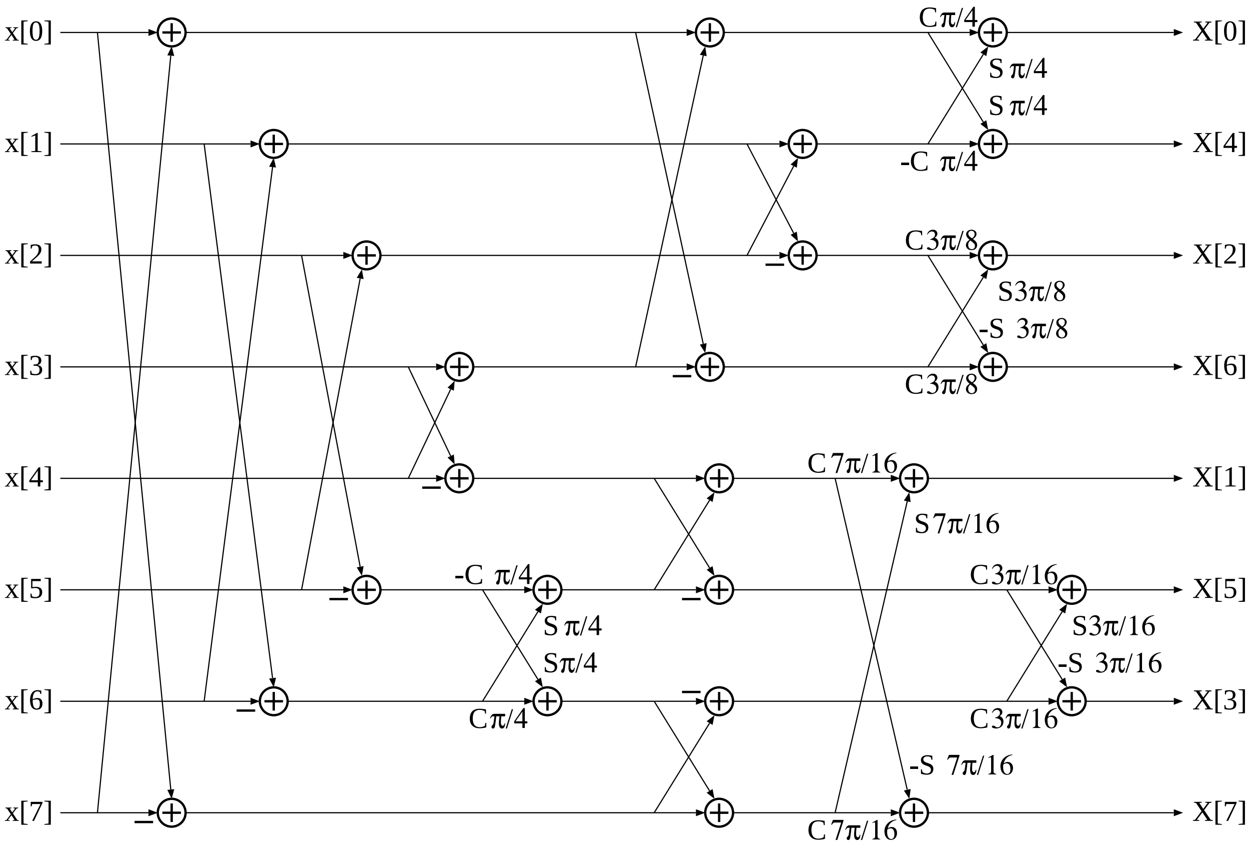

Figure 1: The fast forward DCT.

3. (30 points) Download from the course website *http://thanglong.ece.jhu.edu/Course/643/*:

   - the discrete dyadic wavelet transform (DWT) with 9/7-tap biorthogonal symmetric filters: `wavelet.c` and `iwavelet.c`
   - several popular test images: `barb.pgm`, `lena.pgm`, `goldhill.pgm`, `boat.pgm`.

   In this simple coding experiment, you will investigate and compare transform behaviors under quantization noise. The transformations of interest are the discrete wavelet transform (DWT) with 9/7-tap filters, and the 8×8 block DCT (that you have implemented in the previous problem).

   Write a C program to perform uniform quantization on all transform coefficients such that 90% of the coefficients are quantized to zero. Reconstruct the images and comment on their visual quality. What happens if we reduce the percentage of significant (nonzero quantized) coefficients to only 5%? 1%? What happens if we increase the percentage to 25%? Perform the experiment on at least 3 test images. What transform yields the best visual quality at low bit rates?

Due date: **April 4 in class.**