Homework Assignment I

Reading Assignment: lecture notes, Strang-Nguyen Sections 2.1 – 2.3, 3.1 – 3.3

1. An input signal \( x[n] \) with triangular frequency spectrum is passed through a 2-channel maximally-decimated filter bank with ideal filters as depicted in Figure 1.

   Sketch the signal spectrum at every node. Show graphically that perfect reconstruction can be achieved.

   ![Figure 1: Two-channel filter bank with ideal filters.](image)

2. Let \( G(z) = \sum_{n=0}^{N} g[n] z^{-n} \) be a real-coefficient order-\( N \) FIR low-pass filter whose impulse response is \( g[n] \) and \( g[0], g[N] \neq 0 \). Define the following three filters:

   \[
   H_1(z) = z^{-N} G(z^{-1}); \quad H_2(z) = G(-z); \quad H_3(z) = z^{-N} G(-z^{-1}).
   \]

   (a) Find the impulse responses \( h_1[n], h_2[n], \) and \( h_3[n] \) in term of \( g[n] \).
   (b) If \( g[n] \) is even-length and symmetric, which type of filter is \( h_1[n], h_2[n], \) and \( h_3[n] \)?
   (c) Find the magnitude responses of \( h_1[n], h_2[n], \) and \( h_3[n] \) in term of \( |G(e^{j\omega})| \). If \( G(z) \) is the perfect low-pass filter with cut-off frequency \( \omega_C \), sketch the frequency responses of \( H_1(z), H_2(z), \) and \( H_3(z) \).
   (d) If \( z_0 \) is a zero of \( G(z) \), find the corresponding zeros of \( H_1(z), H_2(z), \) and \( H_3(z) \).

3. Prove that an FIR filter \( H(z) \) of order \( N \) has real coefficients \( h[n] \) if and only if its roots are either real or appear in conjugate pairs. More precisely, the right-hand side means that: if \( z_n \) is a root of \( H(z) \), then either \( z_n \) is real or \( z_n^* \) is also a root of \( H(z) \).

Due date: **Friday September 19** in class