Homework Assignment II

Reading Assignment: Lecture Notes; Strang-Nguyen Chapter 3, Section 4.1-4.2;

1. Suppose that we have four filters $H_0(z), H_1(z), F_0(z),$ and $F_1(z)$ forming a perfect-reconstruction 2-channel filter bank. Investigate the possibility of achieving perfect reconstruction in the following cases.

(a) Interchange $F_i(z)$ with $H_i(z)$, i.e., using the synthesis filters as analysis filters and vice versa.
(b) Modulating all filters, i.e., replacing $H_i(z)$ with $H_i(-z)$ and $F_i(z)$ with $F_i(-z)$.
(c) Delaying the synthesis low-pass filter by $D$, i.e., replacing $H_0(z)$ by $z^{-D}H_0(z)$ and modifying $F_1(z)$ accordingly.

2. Let $P_0(z) = (1 + z^{-1})^6 Q(z)$, find the 4-th degree symmetric polynomial $Q(z)$ that makes $P_0(z)$ halfband. Find and plot all the roots of the resulting halfband filter.

3. Consider the 2-channel filter bank depicted in Figure 1 with

$$H_0(z) = a + bz^{-1} + bz^{-2} + az^{-3}$$
$$H_1(z) = c + dz^{-1} - dz^{-2} - cz^{-3}.$$

(a) Find $F_0(z)$ and $F_1(z)$ such that aliasing can be canceled.
(b) Find the analysis polyphase matrix $H_p(z)$. Prove that $H_p(z)$ has symmetry:

$$H_p(z) = z^{-1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} H_p(z^{-1}) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$  

(c) Find the determinant of $H_p(z)$.
(d) Based on the determinant of $H_p(z)$, find the constraints on $\{a, b, c, d\}$ that yield perfect reconstruction with FIR synthesis filters.
(e) Find the causal FIR synthesis polyphase matrix $F_p(z)$? What are the corresponding synthesis filters? Are your answers consistent with those in Part a?

(f) Imposing one more zero at $\omega = \pi$ on $H_0(z)$ by setting its first derivative to zero at $z = -1$. What are the four resulting filters now?

Due date: **Friday 09/26/2012 in class**