Problem Set VI

1. Demonstrate in Matlab the polynomial capturing ability of the discrete wavelet transform with certain number of vanishing moments. Experiment with your best 9/7 biorthogonal, 6/10 biorthogonal, and 8/8 orthogonal solution obtained from Problem Set IV. What are your observations in the biorthogonal cases?

Now suppose that your polynomial signal is corrupted by white noise. Design a simple algorithm to recover the polynomials.

2. Suppose that we have $K$ vanishing moments in an orthonormal wavelet system, prove that
   \[ \sum_{n} n^k h_1[n] = 0, \quad \text{for } k = 0, 1, \ldots, K - 1. \]
   \[ \int t^k \psi(t) dt = 0, \quad \text{for } k = 0, 1, \ldots, K - 1. \]
   *Hint:* The Fourier transform of $t^k \psi(t)$ is $(-j)^k \Phi^{(k)}(\omega)$.

3. Suppose that $H_0(e^{j\omega})$ is an ideal low-pass filter and $H_1(e^{j\omega})$ is an ideal high-pass filter. Both filters have cut-off frequency at $\omega = \frac{\pi}{2}$ and are periodic with period $2\pi$.
   Find the frequency and time responses of the resulting scaling and the wavelet function, i.e., $\Phi(\omega)$, $\Psi(\omega)$, $\phi(t)$, $\psi(t)$.

Due date: April 8 in class