Department of Electrical and Computer Engineering
The Johns Hopkins University
520.646 Wavelets and Filter Banks – Fall 2018

Problem Set VI

1. Consider a multiresolution analysis and the two-scale equation for $\phi(t)$ given in $\phi(t) = \sqrt{2}\sum_n h_0[n]\phi(2t-n)$. Assume that $\{\phi(t-n)\}$ is an orthonormal basis for $\mathcal{V}_0$ and the associated orthogonal filter bank with $h_0[n]$ and $h_1[n]$. Assume further that $0 < |\Phi(0)| < \infty$ and that $\Phi(\omega)$ is continuous at $\omega = 0$. Prove the following statements.

   (a) $\Phi(0) = \int \phi(t)dt = 1$.

   (b) $h_1[n] = \sqrt{2} < \phi(2t-n), \psi(t) >$.

   (c) $h_0[n] = \sqrt{2} < \phi(2t-n), \phi(t) >$.

   (d) $|\Phi(\omega)|^2 + |\Psi(\omega)|^2 \ = \ |\Phi(\frac{\omega}{2})|^2$.

   (e) $||h_0[n]|| = 1$. *Hint: Start with* $||\phi(t)||^2$.

2. Demonstrate in Matlab the polynomial capturing ability of the discrete wavelet transform with certain number of vanishing moments. Experiment with your best 9/7 biorthogonal, 6/10 biorthogonal, and 8/8 orthogonal solution obtained from Problem Set IV. What are your observations in the biorthogonal cases?

   Now suppose that your polynomial signal is corrupted by white noise. Design a simple algorithm to recover the polynomials.

Due date: **Friday Nov. 2** in class