1. **Recovery Challenge:** I have a signal $x$ of 100 samples ($N = 100$) where only 3 of these samples are nonzero ($S = 3$). The location and magnitude of these nonzero samples are unknown. I used two different sensing matrices on $x$ and obtained two set of measurements. You can download these sensing matrices and measurement vectors from the course web page. Can you recover $x$ by solving the classic $\ell_0$-minimization problem? *Hint:* Matlab built-in function `nchoosek(1 : N, S)` yields all subsets of $s$ columns.

2. Suppose that we now have a longer signal $x$ of length 256 samples ($N = 256$) where 5 of these samples are nonzero ($S = 5$). Again, the location and magnitude of these nonzero samples are unknown. Let us investigate the problem of sampling this sparse signal using the various sampling methods and recover our original signal via greedy pursuit strategies.

(a) Generate the signal using the following Matlab commands:

```matlab
>> x = zeros(N, 1); q = randperm(N); x(q(1 : S)) = randn(S, 1);
```

(b) Does your direct $\ell_0$-minimization algorithm above still work?

(c) Implement the Orthogonal Matching Pursuit (OMP) and Subspace Pursuit (SP) algorithms to solve the following $\ell_0$-minimization problem in a greedy fashion

$$\hat{x} = \text{argmin}_x ||x||_0 \text{ subject to } y = Ax.$$  

For each sensing matrix $A$ itemized below, vary the value of $M$ (say $M = \{10, 20, 30, 40, 50\}$), and perform $\ell_0$-minimization at 50 different instances of the signal $x$ by varying the location and magnitude of its nonzero samples. If $||\hat{x} - x||_2 \leq 10^{-10}$, then we regard the signal recovery as perfect. Plot the performance curve in which $x$-axis represents the number of measurements $M$ while $y$-axis denotes the probability of perfect signal recovery. At each of the 50 instances, the signal should be different and often times, the sensing matrices are different as well. The following sampling schemes/sensing matrices are under consideration.
(a) *Random sampling in the time domain:* Suppose $I$ is the $N \times N$ identity matrix. Create the sensing matrix $A$ by keeping $M$ rows of $I$ at random locations (and deleting the remaining $M - N$ rows).

(b) *Random sampling in the frequency domain:* Suppose $F$ is the $N \times N$ DCT matrix ($>> F = dct(eye(N));$). Create the sensing matrix $A$ by keeping $M$ rows of $F$ at random locations (and deleting the remaining $M - N$ rows).

(c) *Sampling with a random matrix:* The sensing matrix $A \in \mathbb{R}^{M \times N}$ in this case is generated from a collection of random Gaussian variables, then the rows are orthonormalized, i.e.,

$$>> A = randn(M, N); A = orth(A');$$

Which sensing matrices are best for perfect recovery? In those cases, how many measurements are sufficient for perfect signal recovery? Which sampling method seems to be most efficient?

3. Repeat the experiments for the case where the signal is frequency-sparse, i.e., $x$ only contains $S = 5$ significant frequency components. Like the time-sparse case earlier, the location as well as the magnitude of those $S$ frequencies are unknown. Such a signal can be generated as

$$>> alpha = zeros(N, 1); q = randperm(N); alpha(q(1 : S)) = randn(S, 1); x = idct(alpha);$$

What are your observations this time?

Due date: Thurs, Feb. 12th in class